SYNOPSIS

The last two decades have witnessed the advancement of shell analysis by finite element method. Prior to this, analysis of shells had been carried out by series solutions[19,26,45,83,144]* . Direct stiffness method was used by some investigators[87,107] while matrix displacement method[135,163] was also applied for the analysis. Triangular elements were adopted in some cases[54,55,98,109,111,118] ; on the other hand, rectangular shell elements were also chosen by many researchers[49,63] . Primarily, rectangular shell elements contained single curvature and was further generalised for double curvature by several authors[147,181] . The displacement parameters were approximated with the polynomial terms or one-dimensional, first-order Hermite interpolation formulae[30,147,181] . The inclusion of rigid-body modes was studied[41,147,181] by performing eigenvalue analysis with element stiffness matrix. Experimental results were also presented by some authors[84,151] . The dynamic analysis[1,183] , non-linear analysis[52,126] and hybrid stress finite element analysis[180] etc. are some of the important areas which found place in this increasingly popular field of analysis.

The author has attempted to furnish a generalised formulation for the doubly curved shells including all three

*Superior number indicates the reference given at the end of thesis.

radii of curvature using finite element method. To facilitate easy comparison of the results of the shell characteristics viz., stress resultants, stress couples and displacement components as obtained by earlier investigators on specific problems, the author has also taken the same problems for the purpose of numerical computation with the help of his generalised formulation. Numerical results under three different grid sizes have been considered to study their relative accuracy and convergence. The effects of Poisson's ratio (ν) on the shell characteristics have also been studied by considering the results under three different values of Poisson's ratio. The modifications in boundary conditions have been carried out to study the savings in CPU time and computer memory space to get acceptable though approximate design values of shell actions.

In this investigation five numerical problems have been selected for illustration. Problems 1 and 2 (marked as EIGEN - I and II) are concerned with the testing of rigid-body modes of two isotropic shell elements while problems 3, 4 and 5 (defined as PAR-I, HYP-I and HYP-II respectively) have been taken up to study different aspects as mentioned above. The shell elements of problems 1 and 2 were earlier investigated by Yang[181] and subsequently by Rao[147]. Problem 3 is a clamped paraboloid of revolution shell solved earlier by Ramaswamy[144] and later on by Bandyopadhyay[19]. Problem 4 is a square hyperbolic paraboloid shell solved earlier by Chetty and Tottenham[45], Iyengar and

Srinivasan[83], Bonnes[33], Dhatt[54,55], Bhattacharya[26], Choi and Schnobrich[46], Wolf[180], Minich and Chamis[109], Laursen et al.[98] and Bandyopadhyay[19]. Problem 5 is also a square hyperbolic paraboloid shell solved earlier by Connor and Brebbia[49] and Yang[181].

The thesis has been broadly divided into seven chapters and two appendices. The introduction has been put forward in Chapter I, with clear indication of the importance of the problem under investigation from both research and design point of view.

In order to arrive at a definite scope of the present investigation, it is necessary to review the existing literature. An account of the historical survey with greater emphasis on works relating to finite element method, along with a detailed review of some of the investigations has been furnished in Chapter II of the thesis. At the end of the detailed review, a critical discussion on the findings of the various investigators has been made. This has helped the author to define the scope of the present investigation in proper terms. This has been reported in Chapter III of the thesis.

In Chapter IV, the generalised formulation for the doubly curved shells has been attempted using finite element method. For this purpose a doubly curved, four-node, rectangular element, having twelve degrees of freedom at each node has been considered. Each of the displacement parameters u, v and w has been approximated with sixteen term polynomials. Element equilibrium has been considered using the principle of minimum potential energy. Implicit inclusion of six rigid-body modes has been tested by performing eigenvalue analysis on the element stiffness matrix. Nodal displacements and stresses have been computed for normal uniform loading after obtaining global stiffness matrix and corresponding load vector with appropriate imposition of boundary condition. For this purpose necessary computer programmes have been developed.

The author's formulation has been applied to shell problems of both positive and negative Gaussian curvatures after preparing necessary data of the respective problems. The details of the specific numerical problems including the physical dimensions, material properties and loadings etc. have been reported in Section 5.2 of Chapter V . The numerical computations have been performed on a quadrant of the shell surface using three different grid sizes and three different values of Poisson's ratio. All the computations have been performed in double precision using BURROUGHS 6700 computer available at the Regional Computer Centre, Jadavpur, Calcutta. The numerical results, so obtained for shell actions, have been furnished in the form of graphs and tables. In order to facilitate easy comparison, the available results of the earlier investigators have also been presented suitably along with those of the author. All the results (both tabular and graphical) have been furnished in Section 5.3 .

The comparative analysis and discussion of the numerical results of various problems have been reported in Chapter VI . The comprehensive study can be broadly classified under three major aspects. First, the results of the eigenvalues obtained using the author's formulation have been compared with those of others to check proper inclusion of six rigid-body modes for problems EIGEN-I and II . This has been discussed in Section 6.2 . Secondly, in Section 6.3, the author's results have been compared with those of earlier investigators. This includes the comparison of the shell characteristics both in their absolute and percentage forms for paraboloid of revolution and hyperbolic paraboloid shells i.e. problems PAR-I and HYP-I . Thirdly, the author's own results have been taken up for studying the convergence of the results and the effects of Poisson's ratio. The convergence of the author's results have been studied with three different grid sizes viz., 4 x 4, 5 x 5 and 6 x 6 respectively with a particular value of Poisson's ratio for both paraboloid of revolution and hyperbolic paraboloid shell problems as mentioned above. This has been reported in Section 6.4 . In Section 6.5 the investigation on the effects of Poisson's ratio has been made for three different values of ν and with three different grid sizes for both paraboloid of revolution and hyperbolic paraboloid shells i.e. problems PAR-I and HYP-I respectively. Furthermore, in order to save the CPU time and computer memory space, problems PAR-I and HYP-II have been solved with two different boundary conditions. The relevant discussions of the effects of such boundary conditions have been reported in Section 6.6. The overall summary of the comparative analysis and discussion of the results has been reported in Sections 6.7.1 , 2 and 3 of Chapter VI.

In Section 7.2 , a summary of conclusions within the defined scope of the present investigation has been furnished.

In addition, the thesis contains two appendices. The detailed expressions for the elements of the stiffness matrix obtained under the author's formulation have been reported in Appendix A .

In Appendix B, the modules of the computer programmes developed to evaluate the eigenvalues and to determine different shell actions have been reported.

A list of references has been furnished at the end of the thesis.