SCOPE OF THE THESIS

The methods available in engineering literature for studying the behaviour (stability, characteristic roots, stability boundaries and mode boundaries) of periodic systems can be broadly classified under four categories :

(i) Perturbation methods[33,34]

(ii) Methods using Liapunov theory[9,23,24]

(iii) State Transition Matrix (STM) methods[11,17,34]

(iv) Infinite Determinant (ID) methods[2,8,15,16,25,35,37,38]

Methods (i) and (ii) have the following limitations. Perturbation methods can be used for systems having small parameters only, while methods using Liapunov theory are limited by the ability to find suitable Liapunov functions. Also, the latter approach can only determine qualitatively the behaviour of systems at large, and not the exact quantitative response.[34]

Methods (iii) and (iv) are based on Floquet theory. Of these, the STM methods are essentially numerical techniques for determining the state transition matrix of a system over the interval 0 to T, T being the period of the periodic parameters. STM methods have found wide acceptance in literature since they can be applied without any restriction regarding the order of the system and magnitude of the parameters. On the other hand, although the ID formulation (first proposed by Hill for 2nd order systems) is general in nature and not based on any limiting assumption, the methods that are presently available in literature are mainly restricted to 2nd order systems having small periodic parameters.

The main objective of this thesis is to develop ID methods that can be applied to periodic systems irrespective of their order and magnitude of the parameters. With this in mind, the ID approach is considered afresh in this thesis, and arising out of this, three methods have been developed for analysing the behaviour of periodic systems via its ID. These are :

(i) Δ-Method,

(ii) Residue Method,

(iii) Direct Method.

Another aspect that has so far limited the application of ID methods to periodic systems is the computational difficulty. This aspect has been given due emphasis in this thesis and computationally viable methods proposed.

The thesis is divided into five chapters, the development being along the following lines :

Chapter I. A critical review of the ID methods available at present is undertaken in this chapter. First, the formulation of the 'even' ID equation and its subsequent reduction into a transcendental form (as done by Hill[16] for 2nd order systems) has been discussed in some detail. The alternate 'Odd' ID formulation is given next and an equivalent transcendental expression of this odd equation has also been obtained. The even and odd ID formulations, however, lead to identical results (except for a definite imaginary shift) and, as such, a conclusion is drawn that the odd formulation can also be used with equal effect. Next, the ID methods developed by Bolotin[2], and Hayashi[15] are examined. Bolotin's method proposes to give directly the stability boundaries of 2nd order systems in suitable parameter planes. Again, the method of Hayashi (referred to in this thesis as the Δ-method) proposes to analyse stability by checking the signs of the even and odd IDs evaluated at the origin. A careful examination, however, reveals that these two methods are equivalent to each other in that one can be derived from the other. Indeed, the stability criteria as used by these methods are derivable from Hill's equation also, and thus, all the methods available for 2nd order systems are really equivalent to one another.

Next, the ID methods available for 3rd and higher order systems are examined. It is seen that Hayashi or Bolotin's method (developed originally for 2nd order systems) do not yield reliable results in all (higher order) cases. Further, an examination of the work of Lindh and Likins[25] (who proposed to improve upon the shortcomings of Bolotin's method for higher order systems) shows that this method is also unsatisfactory because, apart from being computationally cumbersome, it is prone to yield false stability boundaries in some cases. Further, the method of Cooley, Clark and Buckner[8], which is perhaps the earliest method available in literature to analyse the stability of periodic systems directly from the roots of the ID, is also found to yield erroneous results in certain cases. Next, a critical examination also reveals certain errors in the hypothesis leading to the SR method of Padmanabhan[35] (which, nevertheless, is a great improvement on the method of Cooley et al). Lastly, the works of Richards and Miller[37], and Richards and Cristaudo[38], who obtained a transcendental equivalent of the ID equation, have also been reviewed. It is found that no attempt has been made by these authors to solve this equation. In fact, these authors have used this transcendental equation only for obtaining certain small pumping characteristics of the mode and stability boundaries.

The review indicates clearly that the current status of the methods using ID approach is that they can be used safely for 2nd order systems only, and that, in order to put this approach on a rigorous footing, there is a need to develop analytical and numerical techniques for application of this approach to yield satisfactory results for higher order systems.

Chapter II. In this chapter, the basic philosophy underlying the method of Hayashi[15] has been re-examined and a new approach - one that utilises the knowledge of the modes and the corresponding root patterns of periodic systems of different order - has been developed. This approach yields the same stability conditions for 2nd order systems as developed by Hayashi using the harmonic balance analysis. This approach also naturally leads to a set of three Δ-conditions that are necessary and sufficient for the stability of 3rd order periodic systems. These conditions have been used to find the stability boundaries of a system first considered by Cooley et al; such boundaries are found to tally exactly with the ones obtained using analog computer simulation. Stability analysis of 4th and higher order systems has also been attempted and it is found that although it is not possible to devise Δ-conditions that can conclusively analyse their stability, it is possible to obtain certain necessary conditions for the stability of higher order systems.

Chapter III. It has been mentioned earlier that Richards and Miller[37], and later Richards and Cristaudo[38], have obtained a transcendental equivalent of the ID equation of higher order periodic systems. So far no attempt has been made to solve this TE and obtain the system CEs presumably because of the following reasons :

(i) reduction of the ID equation to transcendental form requires the evaluation of certain residues of the ID, and

(ii) assuming that the residues are evaluated, the solution of the transcendental equation is still quite difficult from the computational point of view.

In this chapter, a critical examination of the nature of the TE (i.e., transcendental equation) is first undertaken which reveals that in order to solve the TE it is essential to evaluate the residues of the system ID at certain poles (which are introduced in the ID to make it convergent). Since the residues of infinite determinants cannot be exactly evaluated, the main problem reduces itself to one of evaluating the same using truncations. Before undertaking this exercise with any degree of confidence, it is necessary to show that (i) the residues obtained using ID truncations converge to the exact ID residues, and (ii) the roots of the TE formed using the approximate values of the residues (obtained using ID truncations) converge to the exact system CEs as the truncation size is increased to infinity. With this in view, it is first shown in this chapter that the residues can themselves be expressed in the form of infinite determinants that converge, and, consequently, can be evaluated to any required degree of accuracy by considering sufficiently large truncations. A minimum truncation size necessary for proper convergence in the residues has also been obtained. Next, in order to prove the convergence of the roots of the TE, the TE has been recast into a simple polynomial form whose coefficients are linear functions of the residues. It is then proved that the roots of this polynomial equation converge to the exact system roots as its coefficients (linear combination of residues) are evaluated using progressively increasing truncations.

It follows therefore that it should be possible to evaluate the characteristic roots as follows : (a) evaluate the ID residues using progressively increasing ID truncations,

(b) obtain the coefficients of the equivalent polynomial equation using the above approximate residue values and (c) solve the polynomial equation. A further examination of the process of reduction of the ID equation to the TE form, however, reveals that, although they have so far been chosen in only one fashion in literature, it is possible to choose the ID poles (that are introduced to make the ID convergent) in an infinitely many ways. Two extreme cases arise in this connection, namely, the 'closed-loop' (CL) choice (as made in literature), and the 'open-loop' (OL) choice of poles. It is seen that, while the CL-choice of poles is obviously the best for systems in which the magnitude of the periodic parameter tends to zero (thus resulting in a dc system), the OL-choice of poles yields results in one-shot for systems having ideal samplers (whose dc, 1st and all higher order Fourier coefficients are of equal strength). Now, since all other types of pumpings are intermediate between the above two cases, it is possible that there may be some optimum choice of poles, intermediate between the OL and CL choice, that would effect the fastest convergence in results for systems with such pumpings. To investigate this aspect further, a numerical experimentation, involving some rectangular pumping systems, has been conducted; the results, however, indicate that, in general, the CL-choice of poles is to be preferred.

Finally, CEs of several systems have been obtained using the CL-residue approach and compared with the same obtained using the STM method. For the same degree of accuracy, this method is found to be computationally faster than the STM method for 4th and higher order systems.

Chapter IV : In this chapter, the problem of evaluation of the CEs of a periodic system directly from the roots of its ID truncations is taken up afresh. Arising out of this, it is shown that

(i) beyond a certain order of truncation only n roots (of the ID truncations) lie in any 'strip' in the complex plane, and

(ii) these n roots converge to the exact CEs of the system as the truncation size is increased to infinity[47].

On the basis of these results a method of obtaining the CEs has been proposed. Further, efficient numerical methods for evaluating the roots of the ID trucations have also been developed.

It is well known that the root pattern of the ID of an nth order system is n-periodic in nature, i.e., (i) the ID has n roots in each of the horizontal strips of width 2w (2w being the pumping frequency) bounded by the lines $\mathrm{Im}(z) = (2l \pm 1)w$, $l = 0$ (the 0th strip), ± 1 (the ± 1th strips), ± 2, ..., and (ii) the n roots in any strip differ from those in the next (above or below) by $\pm j2w$. This means that the roots in any one strip are the basic roots of the system; and if they are known, the complete ID root pattern is automatically obtained.

Now, (except for systems with very small parameters and high frequency pumping) the roots of lower order ID truncations (1x1, 3x3, ...) occur too randomly to yield any coherent information regarding the system CEs. However, from a physical point of view it appears logical that the root patterns of sufficiently higher order ID truncations should approximately be n-periodic in nature, and, indeed, should converge to the ID root pattern as the truncation size is increased to infinity. If this be so, it should then be possible to obtain the *n* basic CEs of a system by evaluating the roots of its progressively increasing order of ID truncations and checking for the settling and convergence of *n* of these roots in any of the infinite number of strips. One of the main contributions of this chapter is the presentation of a formal proof justifying this intuitive approach. Using Hurwitz theorem[48] it is proved that, corresponding to every strip in the complex CE-plane, there exists an order beyond which the ID truncations continue to have exactly *n* roots in that strip, and that these *n* roots converge to the exact CEs of the system as the truncation size is increased to infinity.

In order to study the settling and convergence characteristics of the truncated ID roots, a numerical experimentation (NE), involving several systems, has been conducted. Results show that the roots in the 0th strip converge fastest which could as well be inferred from a physical point of view. What is more significant, the NE reveals that the minimum truncation size that one needs go up to for definite settling as well as sufficient convergence of n 0th strip roots is the one in which the 0th strip roots are just repeated, near-periodically, in the ± 1th strips.

Lastly, two numerical techniques (which made the NE possible) for extraction of the roots of ID truncations have been presented in this chapter. The first, which is applicable for systems with cisoidal pumping, is an efficient algorithm for numerically expanding a truncated ID equation into a polynomial form, the evaluation of whose roots is easy. The second method, which is very general in nature, considers the systems in the state variable form and obtains the ID equations such that the evaluation of their roots becomes an eigenvalue problem. Computationally, the method for cisoidal pumping systems has been found to be much faster than the STM methods, while the general is on a par.

Chapter V : The methods developed in the preceding chapters are capable of analysing periodic systems having specific parameter values. In this chapter, the behaviour of such systems when the parameters are varied is studied using a root-locus approach, attention being confined to phase and frequency coherent systems with cisoidal pumping which may also be represented in the form of a control system that has a plant $q(D)/p(D)$ and a loop-gain of $(K_0 + 2K\cos 2wt)$.

Of the three parameters involved in the loop-gain (namely, K_0, K and w), it is felt that K is of greatest importance, and the loci of the CEs of several 'minimum' plant configurations for variation in K (with w = 1 and K_o = 0) have been studied. (Since it is possible to normalize a periodic system with respect to frequency, unity w systems are only considered. Again, since the effect of variation in K_o be conveniently taken care of by considering different plant configurations, K_o is taken to be zero.) Such loci have been termed as the K-loci of periodic systems. The plant configurations considered are :

(i) <u>2-pole case</u> : The Mathieu system, which in the control form has a plant $1/(D^2+K_o)$ and a loop gain of 2Kcos2t, is the kernel 2-pole system. That is, the CEs of any 2-pole plant can be obtained if the K-root loci of this plant, with K_o taking different values, are known. Such loci for some values of K_o have been numerically evaluated.

(ii) <u>3-pole case</u> : The kernel 3-pole plant in normalized form is $1/ D(D+a)(D-a)+K_o$, i.e., if the K-root loci of this plant for different values of (a, K_o) are known, the CEs of any 3-pole system can be obtained. For convenience in presenting the results, however, the root loci of the $1/(D+2b\cot\theta)(D-b\cot\theta+jb)(D-b\cot\theta-jb)$ plant, for some values of b and θ , have been evaluated. The (a, K_o) to (b, θ) and the reverse transformations have been obtained.

(iii) <u>2-pole 1-zero case</u> : $D/(D^2+K_oD+a)$ is the normalized kernel 2-pole 1-zero plant, and if its K-root loci for differe values of 'a' and K_o are known, the CEs of any 2-pole

1-zero system can be obtained. For convenience, however, this plant has been recast in the $D/(D+b\cos\theta+jb\sin\theta)(D+b\cos\theta-jb\sin\theta)$ form, and the root loci for different values of b and θ have been evaluated.

(iv) <u>4-pole case</u> : Compared to 3-pole or 2-pole 1-zero cases, 4-pole plants have an additional degree of freedom. This means that the complete normalized K-root lociof 4-pole systems would have three variable parameters. In this chapter, however, the K-root loci of the plant $1/(D^4+K_o)$, for different values of K_o, have only been obtained. It is seen that these loci provide an understanding of the behaviour of general 4-pole systems as well.

(v) <u>3-pole 1-zero case</u> : 3-pole 1-zero plants also have got three degrees of freedom, and their normalized K-root loci will have three variable parameters. However, the root loci of the plants $D/(D^3+K_o)$ have only been studied.

In order to predict exactly the small pumping behaviour of periodic systems, an analytical method, involving the 3x3 ID truncations, has been developed. In addition, in order to study qualitatively the behaviour of the root loci for large values of K (without taking recourse to involved numerical calculations), an approximate method, involving the higher order ID truncations in a certain simplified form, has been proposed. Arising out of a study of the root loci of several systems using these methods, some interesting properties of the root lociof general n-pole m-zero systems have been observed.

Several examples have been considered in this chapter to demonstrate the use of the above properties in quickly ascertaining the approximate nature of the root loci of periodic systems, and, consequently, in obtaining their stability boundaries.

It is believed that the thesis contributes to a clear understanding of the ID approach, and establishes it as one of the most effective methods for analysing periodic systems. Moreover, the root-locus study undertaken provides a physical picture of the behaviour of such systems for variations in the parameters.