SYNOPSIS

The inadequacy of classical Newtonian theory to explain the behaviour of many common materials such as plastics, paints, colloidal suspensions, high polymer solutions etc., was realized long back. During the last two and a half decades several attempts were made to modify this theory so that it could be applied to these materials. In 1946, Fröhlich and Sack tried to explain the behaviour of dilute solutions of colloidals and suspensions by assuming that they consist of a large number of elastic spheres distributed uniformly in a viscous liquid. These authors deduced theoretically that such a system, at low rates of shear, is described by the constitutive equation:

$$p'_{ik} + \lambda_1 \frac{\partial}{\partial t} p'_{ik} = 2\eta_0 \left(e_{ik} + \lambda_2 \frac{\partial}{\partial t} e_{ik} \right), \tag{1}$$

where p'_{ik} is the deviatoric stress tensor, e_{ik} the rate-of-strain tensor, λ_1 the stress relaxation time, λ_2 the rate-of-strain retardation time and η_0 the coefficient of viscosity of the dispersed system. In 1958, Oldroyd proposed an universally valid generalization of equation (1) so that it could be applied to three dimensional problems, as

$$\begin{aligned} p'_{ik} + \lambda_1 \left(\frac{\mathcal{D}}{\mathcal{D}t} p'_{ik} - p'_{ij} e_{jk} - p'_{kj} e_{ij} \right) + \mu_0 p'_{jj} e_{ik} + \nu_1 p'_{jl} e_{jl} \delta_{ik} \\ = 2\eta_0 \left[e_{ik} + \lambda_2 \left(\frac{\mathcal{D}}{\mathcal{D}t} e_{ik} - 2 e_{ij} e_{jk} \right) + \nu_2 e_{jl} e_{jl} \delta_{ik} \right], \end{aligned} \tag{2}$$

$$\frac{\mathcal{D}}{\mathcal{D}t} b_{ik} = \frac{\partial}{\partial t} b_{ik} + v_j b_{ik,j} + \omega_{ij} b_{jk} + \omega_{kj} b_{ij}, \tag{3}$$

where μ_0 , ν_1 and ν_2 are material constants each having dimensions of time, b_{ik} any covariant tensor, ω_{ik} $[=\frac{1}{2}(v_{k,i}-v_{i,k})]$ the vorticity tensor, v_k the velocity vector and a suffix following a comma denotes covariant differentiation. Oldroyd (1958) predicted that the liquids governed by the constitutive equation (2) exhibit qualitatively most of the essential features observed in non-Newtonian fluids. In this thesis we have discussed few boundary value problems using the constitutive equation (2), which reveal various aspects of the liquid. It comprises of seven chapters.

Chapter I is of introductory nature and consists of a brief survey of the subject. Starting from the inadequacy of classical Newtonian theory, various ways of generalizing it which lead to constitutive equation of elastico-viscous liquids, are given. A brief account of the experimental work done in elastico-viscous liquids is also presented.

In Chapter II, the axi-symmetric flow near a stagnation point, when the main stream outside the boundary layer oscillates in magnitude (but not in direction) about a steady non-zero mean, is discussed. The analysis is based on Lighthill's theory of expressing the flow functions as the sum of a steady and a small superimposed oscillating part and separate solutions are obtained for low and high frequency oscillations. It has been found that both the amplitude and phase-lead of skin-friction oscillations decrease with increase in elasticity of the liquid. In the high frequency range, the phase-lead of $\pi/4$ for the Newtonian case tends to $-\pi/2$ (a phase-lag) for the elastic case.

The flow induced due to a disk oscillating in its own plane about a steady non-zero mean is discussed in Chapter III. The problem is treated by Lighthill's method and higher-order approximations are obtained for the velocity components in the high frequency range. It is found that both the transverse and the radial velocity oscillations anticipate a phase-lag over the disk-oscillations, near the disk. The oscillating part of the tangential shearing stress at the disk has a phase-lead whereas the oscillating part of the radial shearing stress has a phase-lag over the disk oscillations. The effect of elasticity in the liquid is to increase the phase-lead in the former case and the phase-lag in the latter case.

Chapter IV is devoted to the study of the flow induced between a steadily rotating and a torsionally oscillating disk. The solution is obtained by perturbation method for small values of Reynolds number in infinite series form. It is found that the skin-friction-oscillations on the oscillating disk, has a phase-lead over the disk-oscillations. This phase-lead decreases with the decrease of the distance between the disks and changes into phase-lag when the distance between the disks is less than certain critical value. The flow induced due to a disk oscillating about a steady non-zero mean in presence of another stationary parallel disk, is also discussed.

Chapter V deals with the flow of an elastico-viscous liquid past a sphere. The equations governing the velocity-field within the boundary layer near a body of revolution in a uniform stream are developed. These equations are solved by Kármán-Pohlhausen method using a fifth degree velocity profile. As an example, the flow past a sphere is discussed in detail. The effect of elasticity in the liquid is found to shift the point of separation towards the forward stagnation point and to increase the growth of the boundary layer.

In Chapter VI, the flow induced due to an oscillating sphere when the amplitude of oscillations is small as compared to the radius of sphere, is studied. The elasticity in the liquid results to the formation of a reverse flow in the region close to the surface of the sphere and shifting of entire flow pattern towards the main flow. The effect of elasticity is also found to increase the magnitude of the steady secondary inflow and the resistance on the surface of sphere.

In Chapter VII, we study the natural convection flow of an elastico-viscous liquid past a semi-infinite vertical flat plate maintained at variable wall temperature. A search for similarity solution reveals that such a solution is possible only when the excess of wall temperature over the ambient temperature varies as the first power of the distance from the lowest edge of the plate. The effect of elasticity is to increase the fluid velocity at any point near the plate and decrease it at any point away from the plate. The têmperature at any point in the liquid decreases with elasticity. The skin-friction and the rate of heat transfer from the plate surface are also found to decrease with increase in elasticity of the liquid.

The numerical computation involved in this thesis is performed On an Electronic Digital Computer IBM 1620 installed at the Indian Institute of Technology, Kharagpur.

REFERENCES

1. Fröhlich, H. and Sack, R. (1946) Proc. Roy. Soc. A, 185, 415.
2. Oldroyd, J.G. (1958) Proc. Roy. Soc. A, 245, 278.