CHAPTER 1

GENERAL INTRODUCTION

Given a mapping T of a set X into itself any question which enquires into the existence, nature and number of points x in X such that Tx = x is the subject matter of fixed point theory. The assumptions generally imposed on T and X vary wildly and range practically from none (e.g., X is a set and T is a mapping) to quite stringent ones (e.g., X is a Riemannian manifold and T is an isometry). The fixed point theory as it is to-day has been developed centering mainly around two earlier theorems - one due to Banach (1922) and the other due to Schauder (1930). Both the theorems have rapidly manifested their importance in application to diverse disciplines. This has drawn the attention of many workers who have looked at these theorems from various angles and thereby obtained a number of generalizations. However, because of its analytical simplicity and wide applicability, Banach's contraction mapping principle has drawn more attention than the other. The major portion of the present thesis deals with problems related to contractive type mappings. The converse to Banach's contraction principle, existence of fixed points, structure of the set of fixed points and the convergence of fixed points in relation to convergence of their corresponding mappings have been discussed in this thesis.

The thesis is divided into eight Chapters including the present one which gives a general introduction.

Chapter 2 is concerned with a discussion on Banach's contraction principle and its converses. In dealing with the converses one has to establish the conditions to be satisfied by a mapping so that it becomes contraction with respect to a topologically equivalent metric. The main result here is the derivation of a converse to Jungck's (1976) fixed point theorem in the same spirit as of Meyers (1967).

Chapter 3 deals with the conditional uniform equivalence of orbits introduced by Leader (1977). First, it has been shown here how his fixed point theorem can be generalized by weakening his hypotheses. This has been done by introducing orbital continuity and orbital completeness instead of the completeness of graph used by Leader. Secondly, Leader's theorem has been extended to uniform spaces. Lastly, the concept of uniform equivalence of orbits has been exploited in the study of the convergence of a sequence of mappings in relation to the convergence of the sequence of their fixed points.

In Chapter 4 the concept of positive definiteness is discussed. This idea has been introduced first by Dugundji (1973) in connection with real-valued functions defined over a metric space and has been exploited in establishing fixed point theorems and coincidence theorems. The extension of this concept to uniform spaces has been done by Wong (1974). The main objective of this Chapter is to extend this idea to quasi-uniform spaces. In doing so, we first prove the properties of positive definite functions defined over a quasi-uniform space and then study the properties of completeness and compactness in relation to positive definiteness of real-valued functions. A coincidence theorem has also been given.

Chapter 5 is concerned with a study of the structure of the set of subsequential limit points of a sequence of iterates of a self-map over a metric space. Diaz and Metcalf (1969) have initiated this investigation while extending a result of Tricomi (1916) in connection with iterations of real-valued functions. In this Chapter we first generalize results of Tricomi and Diaz & Metcalf by removing the continuity condition on the mapping throughout the space. Then we study the structure of the set of subsequential limit points of the sequence of iterates of the mappings introduced by Pal and Maiti (1977) and derive the same conclusions as those of Diaz and Metcalf. Finally, we give an example to settle a question in this connection posed by Metcalf and Rogers (1970) in the negative.

The study taken up in Chapter 5 is continued in Chapter 6. Here the metric in the inequality of Diaz and Metcalf (1969) has been replaced by a non-negative real-valued continuous function defined over the product space, and thus their results have been generalized and some related results have been derived. Then this idea has been extended to the case of quasi-nonexpansive and quasi-contractive mappings thereby generalizing the results of

Dotson (1972) and some other results have been derived in the light of a result due to Singh (1973-1974). In a similar fashion the idea has been extended to cover asymptotic regularity and Caristi's (1976) conditions and the same conclusions as those of Diaz and Metcalf have been derived. Next, we generalize the theorems of Diaz and Metcalf by using localized versions of the concepts of orbital continuity and orbital completeness. Lastly, we extend some of the above results to uniform spaces and Hausdorff topological spaces.

In Chapter 7 some results of Petryshyn and Williamson (1973) have been generalized in the spirit of generalizations given in Chapter 6.

Chapter 8 is concerned with the study of fixed points through Cebysev centres. Some results of Reinermann and Schoneberg (1976) have been generalized and in the process a question posed by Naik (1980) has been answered partially in the affirmative.