Abstract

Consider the differential equation

$$-(p(t)y'(t))' = q(t)f(t, y(t), p(t)y'(t)), \qquad 0 < t < 1,$$

where $p(0) = 0$ and $q(t)$ is allowed to be discontinuous at $t = 0$. Such differential equations are called doubly singular differential equation. The boundary condition at the nonsingular end is $y(1) = \alpha_1 y(\eta)$ or $y(1) = \int_0^1 g(s)y(s)\,ds$, under certain conditions on η, α_1, and g. While, the boundary condition at the singular end is decided by the behavior of $p(t)$ and $q(t)$ in the neighborhood of singular point $t = 0$. In case, $1/p \in L^1(0, 1)$, we consider $y(0) = 0$ or $\lim_{t\to 0^+} p(t)y'(t) = 0$ and for $1/p \notin L^1(0, 1)$, we consider $\lim_{t\to 0^+} p(t)y'(t) = 0$.

This Thesis mainly discusses the existence of multiple nonnegative solutions for a class of nonlocal doubly singular boundary value problems. The methodology used to treat such type of boundary value problems has been to write boundary value problem as an integral operator and find a solution as fixed point of equivalent integral operator using well known fixed point theorems.

The thesis consists of seven main chapters and a concluding chapter. First chapter is the introductory chapter which provides introduction, motivation and state of the art for treatment of nonlocal boundary value problems. In second chapter, using Leggett-Williams fixed point theorem, existence and multiplicity of solutions for three-point doubly singular boundary value problem with derivative independent data function f are discussed. While in third chapter, existence of multiple nonnegative solutions for three-point boundary value problem with derivative dependent data function f using fixed point theorem of functional type (generalized Leggett-Williams fixed point theorem) is established.

In fourth chapter, we consider doubly singular nonlocal boundary value problem subjected to integral boundary conditions with derivative independent data function f. Existence and uniqueness of solution is discussed using Banach contraction principle and nonlinear alternative of Leray-Schauder type. In fifth chapter, we extend the work of fourth chapter for nonlocal boundary value problem subjected to integral boundary conditions with derivative dependent data function f.

In sixth chapter, we have used upper and lower solution technique along with quasilinearization and monotonic iterative technique to establish positive solution for the three point boundary value problem. In this chapter, $q(t)f(t, y)$ is allowed to be singular at $y = 0$, $t = 0$ and/or $t = 1$. In seventh chapter, we have derived a monotonic iterative scheme to establish existence of a solution for doubly singular three-point boundary value problem with derivative dependent data function. The scheme is very much useful for the computational purposes.

Keywords: Fixed point theory; Green's function; Singular boundary value problem; Monotone iterative method; Upper and lower solutions