CHAPTER I

GENERAL INTRODUCTION

1.1. Test and measurement error in quality control

Testing of units of a product for judging its acceptability is one of the basic functions of statistical quality control. For judging the acceptability of a product, acceptance tests are carried out on hundred percent inspection or sampling inspection basis. In hundred percent inspection scheme (screening) each unit is individually tested and classed as conforming or non-conforming one according as it meets or fails to meet certain quality specifications. Whereas in sampling inspection scheme (plan) only a few randomly selected units are tested against prescribed specifications. These specifications are generally expressed in terms of one or more important quality characteristics which are likely to be measurable. Since no manufacturing process is good enough to produce all the units of the same quality, some basic variability of the quality characteristic is unavoidable. It is this fact that makes it necessary to specify for acceptability either a lower limit or an upper limit or both, instead of a single value of product quality. The upper and the lower specification limits are determined on the basis of engineering requirements of the product.

Measurements are subject to error: What we actually obtain as an observed measurement of a particular unit, can be regarded as sum of two independent components - one representing the 'true quality of the unit' which remains unchanged from one test-set to another, and the other, the 'measurement error' which varies from one test-set to another. By test-set is meant not only the measuring devices and the procedures specified for their use but also the test personnel. The errors due to lack of precision or due to repeatability of test-set are called test errors.

The test and the measurement errors may cause two types of wrong judgements: (i) Rejecting a unit which actually meets the specifications and (ii) Accepting a unit which actually does not meet the specifications. In order that too many good units might not get rejected and too many bad units might not get accepted, an allowance should be made for inherent test and measurement errors. Eagle (1954) pointed out that the allowance can be made by setting some limits (other than the specification limits) worked out on the basis of variability of measurement error. These limits which he called test limits are thus functions of measurement error variance, but the specification limits, as we know, are the functions of product quality variance.

1.2. <u>Review of previous work</u>

The problem of handling errors of measurement in different test situations has attracted many workers in recent years. Eagle (1954) evaluated the probability of rejecting a conforming unit (the producer's risk) and that of accepting a non-conforming unit (the consumer's risk) for various sizes of measurement errors with the test limits set inside the specification limits+. He computed these risks by numerical integration of areas under the normal curve and presented some graphs, considering a particular case of the specification limits being set at two times the standard deviation of the product distribution. In doing all these, both the product distribution and the error distribution were assumed to be normal and the test and the specification limits to be symmetrical about the mean of the product distribution. Some formulas for computing the producer's and the consumer's risks, requiring the use of Pearson (1931) tables were also presented by him. Wiesen and Clark (1954) and Hayes (1955) extended Eagle's work by working out some more graphs for the producer's and the consumer's risks.

\+ Eagle's terms 'test specification limit' and 'performance specification limit' are synonymous to the terms 'test limit' and 'specification limit' respectively.

Grubbs and Coon (1954) studied the problem of setting test limits relative to specification limits using (i) the criterion of equal risks, (ii) the criterion of minimum sum of risks and (iii) a generalized criterion of minimum weighted sum of risks (the cost of rejecting a conforming unit and the cost of accepting a non-conforming unit being treated as the weights). They prepared extensive tables for the parameters of the test limits relative to the specification limits, assuming the distributions of the product quality and measurement error to be normal. The test limits according to their tables should be set outside the specification limits except for the cases when the cost of accepting a non-conforming unit exceeds six times the cost of rejecting a conforming unit.

Tingey and Merrill (1959) studied a problem similar to that of Grubbs and Coon for asymmetrical test and specification limits, introducing a variable (not constant) loss function in the region of misclassification. The cost of accepting a non-conforming unit was assumed to vary with the degree of non-conformance. They prepared a table of constants to facilitate the construction of test limits relative to the specification limits for the risks associated with the costs of misclassification to be minimum.

As the problem of finding the producer's and the consumer's risks involves the computation of bivariate normal probabilities, Owen and Wiesen (1959) gave a method for computing bivariate normal probabilities and produced some charts for evaluation of the risks. They considered, in addition to previous cases, two more cases - in one, the product is not necessarily centered relative to two-sided specification limits, in the other, only one-sided specification limit is prescribed. The expressions for the parameters which determine the position of the test limits relative to the specification limits were also derived by them in all the cases.

Jackson (1957) studied the effect of inspection errors[+] on waste and on quality of outgoing product considering hundred percent inspection. He assumed the inspection procedure to be qualitative and not quantitative in nature, for as he observed the units are compared with such as feel or appearance and not with quantitative measurements such as weighing or gaging.

Diviney and David (1963) established a relationship between measurement error and product acceptance, considering the error variation to be a substantial part of the observed variation. They derived the true and the observed operating characteristic curves based respectively on the true and the observed fraction defectives. Bennett (1954) studied

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+ Jackson has defined the inspection errors as the error of rejecting an item of acceptable quality and that of passing an item that is defective.

the effect of measurement error on the power of control limits, in cases where the control charts are used for acceptance inspection.

1.3. Problems under investigation

The earlier studies of measurement error mainly consider hundred percent inspection, and assume normality of the distributions of manufactured units and measurement error. The study of the effect of measurement error on the characteristics of the sampling inspection plans appears to have received very little attention. In this investigation the effect of the measurement error on hundred percent inspection programmes as well as on sampling inspection plans has been considered for normal as well as non-normal product distributions. The study has also been extended to a selective assembly problem involving hundred percent testing and classification of units into more than two ordered groups. An allied problem of the effect of non-normality on sequential test for mean with two alternative hypotheses has also been studied.

In Chapter II, formulae for evaluating the probability of rejecting a conforming unit (Producer's risk) and that of accepting a non-conforming unit (Consumer's risk) have been derived on the assumption that the true quality characteristic and the measurement error are normally distributed and that the test and the specification limits are asymmetrical. The results for the case of symmetrical limits as well as for the case of one-sided test and specification limits have been obtained as particular cases. Tables have been prepared for these risks, for different combinations of test and specification limits and with varying amount of measurement error. The important uses of the tables have been described and illustrated by means of suitable examples.

In Chapter III, some of the results of Chapter II, have been extended to the case of non-normal product distribution. Expressions for the producer's and the consumer's risks have been derived under the following assumptions: (i) the non-normal distribution is representable by the first four terms of an Edgeworth series, and (ii) the error is normally distributed with mean zero and a constant standard deviation. For certain combinations of the parameters, the normal theory risks and the corrective factors due to kurtosis have been tabulated. It is seen that for platykurtic populations, if the specification limits are set near the mean, both producer's and consumer's risks are greater than their corresponding normal theory values, and if the specification limits are set far from the mean, both the risks are smaller than their corresponding normal theory values. The situations, however, are the other way round for leptokurtic populations. The main results of this chapter are illustrated graphically by a hypothetical problem.

In Chapter IV, a selective assembly problem has been considered which basically involves the classification of each component in n-groups according to size and then assembling the units in the corresponding groups. For the commonly used case of three classes in selective assembly for cylindrical fits, the optimum class boundaries have been obtained by using separately the 'criterion of minimum rejection level' and the 'criterion of minimum variance' of the clearance. The component dimensions and the measurement error have been assumed to be normally distributed.

In Chapter V, the formulae of computing the probabilities of misclassification due to measurement error have been derived. These probabilities have been used for finding the true operating characteristic and average out going quality curves for a single sampling plan for attributes. An investigation has also been made about the size of the sample required for accepting a lot with desired confidence in the presence of measurement error.

In Chapter VI, the effect of measurement error on the operating characteristic function of a single sampling plan by variables has been considered. The effect has also been considered for the samples coming from a non-normal population represented by the first four terms of an Edgeworth series.

In Chapter VII, we have devoted to study the effect of non-normality on sequential test for mean with two alternative hypothesis by obtaining the operating characteristic function for non-normal populations represented by the first four terms of the Edgeworth series. It is found that the normal theory results can be used for samples coming from a moderately non-normal population.