SYNOPSIS

The flow properties of a fluid regarded as a continuum are determined by the well known equations of momentum, continuity, and the energy equation. To achieve any progress in the study of the flow properties we must specify the relationship between the stress tensor and the other properties of the flow, which are determinable with the help of the equations of motion. By assuming a linear relationship between the stress and the rate of strain tensors, the Newtonian fluids have been extensively studied both theoretically and experimentally. However, the assumed stress-strain velocity relationship limits the applicability of the results of this theory.

In recent years it has been realised that the fundamental stress-strain velocity relations of classical hydrodynamics are not applicable to a vast number of highly viscous liquids like mercury, colloidal suspensions, high polymer solutions etc., which have got great technological importance. The three major phenomena which describe the normal stress effects in the fluids are Weissenberg effect, Poynting effect and Merrington effect. In fact, it is well known that the above three phenomena are the most remarkable contributions to the fluid dynamics which have attracted the attention of many authors to study the normal stress effects. Among them Reiner-Rivlin have proposed a simple and purely hydrodynamical theory by assuming a non-linear stress-strain velocity relationship which now contains two viscosity coefficients. The introduction of the cross-viscosity term in the constitutive equation is responsible for attributing to the fluid the Weissenberg effect, the climbing of the fluid along the inner fixed cylinder when the fluid is sheared between two coaxial cylinders by rotating the outer cylinder, the Poynting effect where if the liquid is sheared between two parallel infinite plates the shearing forces alone are insufficient so that stresses normal to the plates must be applied to obtain the required steady state of the flow, and the Merrington effect, the swelling of the stream of fluid at the exit of the circular pipe through which it has been flowing.

The equations governing the flow of non-Newtonian liquids are highly non-linear and the simplification of the equations is certainly desirable in order to get at-least an approximate solution of these complicated problems.

The work contained in this thesis deals with the aspects of stability of revolving non-Newtonian incompressible liquids, the discussions being restricted to the case of the flows between concentric rotating cylinders. The technique of normal modes has been used throughout the present work to determine the characteristic modes leading to the stability or instability of the hydrodynamic or hydromagnetic system considered. The small gap approximation (the gap between the cylinders is small) which gives us two types of simplifications has also been applied to the problems considered. The main idea of the present investigations is to find whether the non-Newtonian liquids are more stabilising or destabilising in their nature, compared to the classical viscous liquids or in other words to examine whether cross-viscosity acts as a stabilising or destabilizing agent.

The investigations in the present thesis comprise of five chapters. In chapter I, a detailed discussion of the aspects of stability with its historical background, the basic equations of non-Newtonian liquids and the survey of literature on the stability of non-Newtonian liquids have been given. Attempts are also made to discuss clearly the problems investigated in the thesis.

Chapter II deals with the stability of non-Newtonian flow between rotating concentric cylinders in the presence of a transverse pressure gradient. The liquid is flowing between the cylinders and at the same time the fluid is being pumped round the annulus so as to maintain a transverse pressure gradient. The pumping may be in the direction of rotation or opposite to it. The resulting eigenvalue problem has been solved by a general Galerkin's principle. It is found that the critical Taylor number which decides the onset of instability decreases as the cross-viscous parameter S ($\frac{1}{2}\frac{\nu_c R_1}{d^3}$, where ν_c is the Kinematic cross-viscosity, R_1 the radius of the inner cylinder and d is the gap between the cylinders) increases for a fixed K (parameter describing the pressure gradient), which shows that the region of stability is decreased as the cross-viscosity in the fluid is increased.

Chapter III is concerned with the discussion of the rotational instability of a non-Newtonian flow between co-axial cylinders when an axial pressure gradient is present. Taking a first approximation to the solution of the Taylor number T , the values of T have been obtained for different values of the cross-viscous parameter S , and the Reynolds number R (Reynolds with respect to the mean axial flow). It is found that the Taylor number T decreases as S increases for a fixed R , which again shows that cross-viscosity acts as a destabilizing agent. Graphs are also drawn to exhibit the regions of stability for different values of S .

A detailed investigation on the hydromagnetic rotational instability of a non-Newtonian liquid, which is taken to be an electrical conductor, has been made in chapter IV. The magnetic field is assumed to be in the axial direction. It is found that effect of the magnetic field depends on the non-dimensional number

$$Q = \frac{\mu^2 H^2 \sigma}{\rho \nu} \cdot d^2 \quad ,$$

where ρ is the density of the fluid, μ the magnetic permeability, H the strength of the magnetic field, σ the coefficient of electrical conductivity and d is the gap between the cylinders, and the effect of cross-viscosity depends on S . The resulting eigen value problem is solved by the Galerkin's method in both the cases when the walls of the cylinders are perfectly conducting and non-conducting. It is found (in both cases) that the effect of the magnetic field is to inhibit the onset of instability as in the case of classical viscous liquids, the extent of inhibition depending on Q , whereas the effect of cross-viscosity is to destabilise the flow even when a magnetic field is present. Neutral stability curves are drawn to exhibit the two regions of stability, and velocity profiles are also given for different values of Q and S.

In chapter V, a study has been made on the stability of an electrically conducting non-Newtonian liquid with variable density between concentric rotating cylinders when a circular magnetic field is present. A uniform current of intensity j_0 flowing through the fluid in the axial direction and an axial current of total intensity J_0 within the inner cylinder can give rise to a circular magnetic field. Recently, it was shown in the case classical viscous liquids that the fluid cannot remain stable in the whole region when a circular magnetic field is present. It was found that the flow will be destabilized by the magnetic field produced by the system of currents if

$$-\pi R_0 d < \frac{J_0}{j_0} < \pi R_0^2 ,$$

and stabilized if

$$\frac{J_0}{j_0} > \pi R_0^2 , \quad \text{and} \quad \frac{J_0}{j_0} < -\pi R_0 d ,$$

where R_0 is the mean radius of the cylinders. An attempt is made in this chapter to discuss the effect of a circular magnetic field on a non-Newtonian flow. The values of the critical Taylor number T_c which decides the onset of instability are found and it is seen that the circular magnetic field will play the dual role of stability as in the case of classical viscous liquids, and that these roles are more pronounced as the cross-viscous parameter S increases. (i.e. As S is increased the fluid will be more stabilized or destabilized according to the case considered). Further, the results are also shown in graphs to exhibit the regions of stability for different values of the parameters occuring in the problem.

The numerical computation contained in this thesis has been carried out on the Electronic Digital Computer, National Elliot 803, installed at the Hindustan Aircraft Ltd., Bangalore.