

Synopsis

The inadequacy of the classical viscous theory given by Newton was realized long back. Reiner (1) was the first to give a non-linear theory between stress and strain-rate relation to explain a few peculiar phenomena exhibited by highly viscous fluids, colloids, suspensions and solutions of rubber in mineral oils. Later on, Rivlin and Ericksen (2) have modified the constitutive equation suggested by Reiner to explain the inequality of two normal stresses perpendicular to the plane of shear. Even the modified constitutive equation given by them could not explain the stress relaxation effect present in most of the liquids which exhibit normal stress effect. Noll (3) proposed the general definition of fluidity and developed the constitutive equation of a fluid from this general definition and called this fluid to be a simple fluid. This simple fluid is more general than Rivlin-Ericksen fluid. Coleman and Noll (4) constructed the constitutive equation in which all the second-order terms in time scale are taken into account and proposed the following equation for a second-order fluid,

$$p_{ij} = -p\delta_{ij} + \mu_1 A^{(1)}_{ij} + \mu_2 A^{(2)}_{ij} + \mu_3 A^{(1)}_{ik} A^{(1)}_{kj},$$

$$A^{(1)}_{ij} = v_{i,j} + v_{j,i}$$

$$A^{(2)}_{ij} = a_{i,j} + a_{j,i} + 2v_{m,i} v_{m,j}$$

where p_{ij} , is the stress tensor, v_i is the velocity vector, a_i is the acceleration vector, μ_1, μ_2, μ_3 are material constants and p is an indeterminate hydrostatic pressure.

In this thesis we have solved a number of boundary value problems which reveal various aspects of this fluid. In the first Chapter the general idea of simple fluids is explained and it is shown how we can get the constitutive equations for first-order fluid (Newtonian-fluid), second-order fluid and fluids of higher order. The significance of the second-order fluid and the existing literature have also been pointed out.

In the second Chapter Ram Ballabh's idea of superposability and self superposability have been extended to second-order fluids. Irrotational motion and Beltrami flows are self superposable and mutually superposable even if the fluid is of second-order. The condition of integrability of Beltrami flow is given.

The forced flow of second-order fluid against a rotating disk has been discussed in the third Chapter. The flow due to rotation of disk (von-Karman's problem) is a particular case and the heat transfer for this case has been solved. The momentum and energy equations have been solved by Karman-Pohlhausen method. The

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normal stress on the plate increases with the increase of (i) the angular velocity of the disk and (ii) the velocity at infinity. The radial shearing stress on the plate decreases due to the second-order terms while the dimensionless moment coefficient increases. The effects of the variation of $\beta (= \frac{a}{\omega})$ and $\gamma = (\frac{\omega}{a})$, where ω is the uniform angular velocity of rotation of the disk and a is a constant depending upon the velocity at infinity, on the boundary layer thickness and velocity profile are qualitatively similar to those in the case of ordinary viscous fluids. As far as heat transfer due to disk-rotation is concerned, the second-order effect is to increase the thermal boundary layer thickness and to reduce the rate of heat transfer on the plate.

The fourth Chapter deals with the flow of second-order fluid confined between two infinite parallel rotating disks. The Non-Newtonian effects are more pronounced when the disks rotate in the opposite sense. The effect of second-order terms in the constitutive equation is to change the force of attraction to that of repulsion when the disks rotate in opposite sense.

The fifth Chapter deals with the secondary flow in a rotating ~~straight~~ straight circular pipe when the axis of rotation is perpendicular to the axis of the

pipe. The second-order terms in the constitutive equation of state does not effect the nature of the stream lines i.e. they are spirals similar to those in the case of Newtonian fluids. An approximate formula for the flux through the pipe is deduced. This formula can be utilised to determine the material constants of this fluid. The Non-Newtonian effect is to increase the resistance coefficient of the pipe.

The sixth Chapter is devoted to the study of the fluctuating flow of a second-order fluid near a stagnation point. The equation's of motion have been solved by using Lighthill's technique. Two approximate solutions for large and small values of the reduced frequency ω ($\approx \frac{\omega}{a_0}$) where ω is the frequency of oscillation and a_0 is a constant depending on the velocity at infinity, have been given. They overlap for a critical matching frequency which increases due to increase of non-Newtonian effect.

REFERENCES

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