

# Abstract

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This thesis examines various mathematical models of Mamdani-type and Takagi-Sugeno-type fuzzy controllers within the framework of fuzzy PI, fuzzy PD, and fuzzy PID controllers. It is imperative to derive explicit mathematical models of fuzzy controllers, as this enables the application of conventional control theory and computational tools to facilitate the analysis, design, and implementation of the controllers in real-time. It also provides a comprehensive understanding of the controller's structure. Mathematical models of eight simplest Mamdani-type fuzzy PI/PD controllers are derived utilizing a two-dimensional input space and Bounded Sum aggregation. Subsequently, the mathematical models of seven simplest Mamdani-type fuzzy PID controllers are presented utilizing a three-dimensional input space and Bounded Sum/Maximum aggregation. Further, in order to look into the development of Takagi-Sugeno-type fuzzy controllers, the models of two simplest three-input fuzzy PID controllers are derived utilizing the Minimum AND operator. The aforementioned fuzzy controllers are obtained with type-1 fuzzy sets only. In the next stage, a general Mamdani-type interval type-2 fuzzy PID controller is developed using a one-dimensional input space and non-uniformly distributed fuzzy sets to explore the capabilities of interval type-2 fuzzy sets. Subsequently, the properties of all of the resulting controllers are examined in detail. Their suitability for control applications is assessed, and their computational aspects are discussed. Finally, to validate the theoretical developments, the applicability and robustness of each proposed controller are demonstrated through simulation studies and real-time experiments on nonlinear plants.

**Keywords:** Mathematical modeling, Mamdani-type controller, Takagi-Sugeno-type controller, Nonlinear control, Fuzzy control, fuzzy PI/PD/PID control, Type-1/Interval type-2 fuzzy sets, Bounded Sum/Maximum aggregation, Weighted average/centre of sums/centre of area defuzzification, Simulation, Real-time implementation.