# Analysis of Functional Data with Structural Anomalies

## Abstract

The primary objective of this thesis is to develop statistical methodologies for identifying and locating structural shifts and anomalies in sequences of densely observed datasets. Structural shifts may occur either in the mean or in the covariance; however, this work focuses exclusively on detecting structural changes in the mean. Representing densely observed data as functional data provides a more informative and coherent framework, but introduces the challenge of infinite dimensionality. This issue is addressed through Functional Principal Component Analysis (FPCA), which projects functional observations onto a finite-dimensional subspace for efficient analysis.

A new data-driven test is proposed for detecting a single change point in independent and identically distributed (i.i.d.) functional data. Unlike existing approaches, the number of principal components is selected dynamically based on data characteristics, enhancing the test's power.

The above idea is extended for estimating multiple change points simultaneously in the mean of i.i.d. functional data without using binary segmentation. The method identifies significant local maxima of a modified CUSUM statistic and effectively distinguishes true change points from insignificant ones.

Then it is generalized to estimate multiple change points in weakly dependent functional data by incorporating a long-run covariance operator. The proposed method accurately detects multiple change points and demonstrates improved performance compared to the binary segmentation method. Changepoint estimation techniques are applied in Cancer Rate, Federal Reserve, Interest Rate, Phoneme and Daily Yield Curve datasets.

Finally a new adaptive, data-driven outlier detection algorithm (STEPOUT) is developed. This two-stage procedure refines covariance estimation iteratively and dynamically selects the number of principal dimensions. Extensive simulation studies demonstrate that STEPOUT achieves superior accuracy and robustness compared to existing methods. The algorithm is also applied to several real-world datasets, including global temperature anomalies, Barcelona rainfall data, and NOx concentration data from Poblenou, Spain, successfully identifying meaningful outliers.

Overall, this thesis contributes novel methodologies for change-point and outlier detection in functional data by integrating adaptive dimension selection and robust statistical estimation.

**Keywords:** *Functional Data Analysis, Change-Point Detection, Outlier Detection, Functional Principal Component Analysis, Weakly Dependent Data, Adaptive Dimension Selection, CUSUM.*