Abstract

Some of the well-known notions of well-posedness are closely related to the convergence of slices, which, in turn, are linked to several interesting geometric properties of Banach spaces. On the other hand, algorithms for single-valued and multi-valued nonexpansive-type mappings have continued to attract considerable attention due to their significant utility and wide applicability in modern mathematics and various applied fields. This thesis examines the geometric properties of generalized locally uniformly rotund Banach spaces, focusing on their structure and characteristics. It also develops some iterative algorithms for finding common solutions involving nonlinear mappings. We divide the study into two parts.

In the first part of this dissertation, we focus on examining the geometric properties of generalized LUR Banach spaces by exploring the interconnections among the properties HLUR, (HS), CLUR, ALUR, and related concepts. We also establish links between the notion of generalized well-posedness in optimization and the rotundity properties of Banach spaces. In addition, we investigate the relationship between near smoothness and the property (HS), and analyze the norm-to-weak upper semi-continuity of pre-dual mappings in relation to the property (HS). Furthermore, we study the concept of a strong slicing constant associated with non-trivial convex subsets of Banach spaces and examine the relationships between condition (S) and the properties (HS) and (HC), which describe the interplay between convexity and slicing behaviour.

The second part of this thesis introduces and studies the concept of ϕ-best proximity points in Banach spaces via the generalized projection operator. We define the ϕ_p-property and the ϕ-proximal property and establish the strong convergence theorems for finding ϕ-best proximity points. Several hybrid and shrinking projection algorithms are proposed to compute common ϕ-best proximity points and solutions to related equilibrium problems in uniformly convex and uniformly smooth Banach spaces. Moreover, we introduce a new nonlinear operator, the (IP)-operator, and develop iterative schemes involving two (IP)-operators to prove weak and strong convergence theorems. These results are further extended to Hilbert and Banach space settings, including applications to split ϕ-best proximity points and variational inequality problems.

Keywords: Locally uniformly rotund (LUR); Property HLUR; CLUR; Property (HS); Generalized well-posedness; Slice; Strong slicing constant; Condition (S); Generalized projection operator; Duality mapping; ϕ-best proximity point; Iterative algorithms; Nonextensive mapping; ϕ_p-property; Uniformly convex; Uniformly smooth; Equilibrium problem; Variational inequality problem; Proximally weakly suppressive mapping; Property (ϕ-UC); (IP)-operator.