# Abstract

The primary objective of this dissertation is to explore how gradients in surface tension influence the motion of droplets and emulsions when subjected to an external flow. Such gradients, which induce Marangoni stresses at the fluid-fluid interface, may arise due to several factors: (i) the presence of a thermal gradient within the flow, (ii) the addition of surface-active agents that are insoluble in the bulk fluid, (iii) a combination of both thermal gradients and surfactants, and (iv) a combined influence of chemical concentration gradients and surfactants in the surrounding fluid. To systematically examine these effects, a comprehensive mathematical model is developed for analyzing Newtonian droplet behavior under low-Reynolds-number (creeping) flows. A major aspect examined is the nonlinear dynamics caused by convective surfactant transport along the droplet interface. This complexity rules out the possibility of treating the combined effects as a straightforward superposition of individual thermocapillary or solutocapillary contributions with the flow-induced behavior, thereby requiring a unified and non-linear analytical approach.

To better align with realistic microfluidic scenarios, Chapter 3 focuses on the behavior of a surfactant-laden droplet containing internal thermal singularities, representative of heat-generating encapsulated microorganisms. These singularities, modeled as monopoles (either centred or off-centred) and dipoles, give rise to additional surface tension gradients at the droplet interface. The analysis reveals that a centred monopole has no effect on droplet migration, while an off-centered monopole or a dipole can substantially alter the migration velocity. Importantly, the interplay between Marangoni stresses, arising from non-uniform surfactant distribution, and internal thermal singularities leads to significant cross-stream migration. Remarkably, this migration occurs even when the droplet is initially positioned on the centreline of an imposed Poiseuille flow, typically considered the most stable location for droplet transport.

In Chapter 4, the influence of flow domain geometry—tubular, or rectangular—on the migration of droplets with internal thermal singularities is investigated under the small surface Péclet number limit. Such configurations are especially relevant to microfluidic applications where flow control is essential. Many biomedical devices rely on parallel-plate setups, which induce combined shear and pressure-driven flows. This study examines how transitioning from a Poiseuille (pressure-driven) to a Couette (shear-driven) profile enables refined control over droplet migration. To ensure a constant volumetric flow rate, two governing flow parameters are defined and mathematically related. These parameters are varied to distinguish between shear-dominant, pressure-dominant, and mixed flow regimes. The droplet's streamwise and cross-stream migration velocities are systematically evaluated, and the corresponding flow fields are analyzed across different parameter combinations. The findings indicate that rectangular channels offer superior control over droplet trajectory compared to cylindrical geometries, making them a more suitable choice for precise droplet steering in microfluidic systems.

Chapter 5 investigates the motion of a droplet subjected to solutocapillary effects in the presence of surfactants, suspended in an imposed Poiseuille flow. The surfactant is assumed to undergo a first-order reaction on the droplet interface, introducing a coupling between interfacial chemistry and hydrodynamics. A regular perturbation expansion in terms of the small surface Péclet number is employed to examine the combined effects of convective transport and interfacial reactions. The analysis reveals that streamwise migration is influenced at the first order in the surface Péclet number, while cross-stream migration emerges only at the second order. Interestingly, an increase in reaction rate enhances the streamwise migration velocity but suppresses cross-stream migration, thereby contributing to droplet stabilization. The flow behavior is further classified into diffusion-dominated and reaction-dominated regimes. Within this framework, optimal parametric combinations are identified that correspond to the minimum and maximum droplet migration velocities.

Chapter 6 examines the migration of a droplet influenced by the combined effects of thermocapillary and surfactant-induced Marangoni stresses in the presence of a nearby bounding wall. To address the wall-induced confinement, the analysis is carried out using a bispherical coordinate system, which is well-suited for capturing interactions between the droplet and the wall. The governing equations are solved order-by-order using a perturbation expansion in the limit of small surface Péclet number. A streamfunction formulation is employed, allowing the velocity and pressure fields to be represented as infinite series. Based on this formulation, a semi-analytical framework is developed to compute the droplet's migration velocity and the surrounding flow field. The findings indicate that the migration velocity increases monotonically with the wall separation distance, highlighting the influence of confinement on droplet dynamics.

Throughout this thesis, the physical insights and underlying reasoning presented aim to advance the scientific and engineering understanding of droplet migration influenced by Marangoni effects under imposed flow conditions. The outcomes demonstrate that droplet dynamics can be effectively regulated by strategically leveraging surface tension gradients. This control mechanism offers a powerful tool for manipulating droplets in microfluidic environments. The results emphasize that the interplay between thermal, chemical, and hydrodynamic forces governs droplet behavior in complex and often nonlinear ways. By incorporating internal thermal sources, reactive interfacial processes, and geometric confinement, the study presents a unified framework that captures the essential physics of droplet migration in confined domains. Collectively, these findings fill critical gaps in the existing literature and offer valuable guidance for the design and optimization of droplet-based microfluidic technologies, with potential applications in diagnostics, targeted drug delivery, and controlled emulsion systems.

***Keywords*** : Droplets, Stokes flow, Marangoni stress, thermocapillary, solutocapillary, internal thermal singularity, Couette-Poiseuille flow, first-order reaction, bispherical coordinates, solenoidal decomposition.