

Abstract

This thesis presents a comprehensive investigation into the mechanisms governing sediment transport and its vertical distribution in suspension within turbulent flow conditions of ice-covered channels. Mathematical models are developed using both the classical advection-diffusion equation (ADE) and the fractional advection-diffusion equation (FADE) to capture the complex interplay between flow turbulence and sediment dynamics. By incorporating key turbulent features and fractional order derivatives, the models offer an improved representation of sediment behavior in ice affected environments.

Unsteady two-dimensional mathematical model have been formulated to investigate the spatial and temporal distribution of suspended sediment concentration in turbulent flow conditions. The governing second order partial differential equation (PDE), representing the advection-diffusion mechanism of sediment transport, has been solved using a numerical method under appropriate initial and boundary conditions. The simulation results reveal that sediment concentration within the suspension region increases consistently in the downstream direction and over time, eventually approaching a quasi steady state. A comprehensive sensitivity analysis has been conducted to examine the influence of key model parameters on sediment distribution.

The phenomenon of non-local mixing, arising from turbulent bursting events, enables sediment particles to exchange across fluid layers and is modeled using an unsteady one-dimensional space-fractional advection-diffusion equation with a spatially variable fractional order θ_1 . This model, developed for sediment transport in ice-covered channels, is numerically solved using the collocation method with shifted Chebyshev polynomials of the first kind. Simulation results indicate that sediment concentration decreases with increasing θ_1 , highlighting the attenuating influence of enhanced non-local mixing. Furthermore, to account for sub-diffusive transport behavior resulting from memory effects, a time fractional ADE with fractional order θ is proposed. An analytical solution is derived via Laplace and Fourier transforms. The results demonstrate that time memory effects significantly decrease the concentration magnitude during the initial stages of transport, gradually diminishing as the system approaches a steady state.

To gain deeper insight into sediment transport dynamics, a coupled analysis of flow velocity and sediment concentration, which are closely linked through particle turbulence interactions, is conducted. A coupled mathematical model is developed for sediment-laden flows, incorporating the effects of stratification. The results reveal that stratification leads to a decrease in sediment concentration due to flow stabilization, while simultaneously increasing flow velocity by suppressing turbulent mixing.

All the previously discussed models in the literature assume a uniform temperature across the water column; whereas natural systems exhibit vertical temperature gradients from the bed to the ice cover. To more accurately represent such conditions, a model is developed incorporating variable temperature. The results demonstrate that thermal stratification induces an inverse relationship between temperature and settling velocity, whereby higher temperatures lead to reduced settling rates.

Keywords : Ice-covered channel; Turbulent flow; Streamwise velocity; Suspended sediment concentration; Stratification effect; Non-local mixing due to turbulent bursting and time memory phenomena.