A Study of Proximity Problems in Graphical and Geometric Settings

Abstract

Doctor of Philosophy

by

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Abstract

The core of this work lies in the theme of *proximity*, which serves as a unifying principle across several domains of graph algorithms, combinatorial optimization, computational geometry, and discrete mathematics. Motivated by applications in monitoring, transportation, and structural properties of graphs, we investigate four problems that exemplify the role of proximity in algorithmic design and complexity analysis.

The first problem, *Minimum Target Coverage for Air Quality Monitoring Using Bus Routes*, arises in urban sensing. Given a set of bus routes and a collection of target locations, the goal is to place sensors on bus routes so that every target is monitored while minimizing the number of sensors. We analyze the computational complexity of this problem, provide approximation algorithms, hardness reductions, and demonstrate its effectiveness in practical air quality monitoring.

The second problem, the *Minsum Problem for Discrete and Weighted Set Flow on Dynamic Path Networks*, extends classical dynamic flow models by incorporating both weighted and discrete constraints. The objective is to minimize the total arrival time of flow units under time-dependent capacities. We first establish hardness results for discrete and weighted set flow problem and then develop efficient approximation techniques for the bin packing problem. In addition, we present approximation algorithms and hardness results for the corresponding dynamic network flow problem.

The third problem, the *Minimum Consistent Subset (MCS)*, is defined on a vertex-colored graph where the task is to select a minimum subset of vertices such that each vertex has a neighbor of the same color within the chosen set, under hop distance. The problem is known to be NP-complete even for planar graphs. We strengthen this result by proving NP-completeness even on trees. On the algorithmic side, we present a fixed-parameter tractable algorithm parameterized by the number of colors. Furthermore, we show that the problem remains NP-complete on interval graphs, thereby extending the landscape of graph classes where MCS is intractable.

The final problem, the *Minimum Selective Subset (MSS)*, was introduced by Wilfong in 1991 alongside the Minimum Consistent Subset problem. Given a vertex-colored graph, the task is to select a minimum subset of vertices such that every vertex v has at least one nearest neighbor of the same color within the graph, disregarding same-colored vertices that are not part of the subset. While the only known result so far was NP-completeness in general graphs, we extend its algorithmic study. We present an approximation algorithm for general graphs, prove NP-completeness even for planar graphs with just two colors, and give linear-time exact algorithms for trees and unit interval graphs.

Together, these contributions advance the understanding of optimization in dynamic networks and graph structures, blending theory with applications in monitoring and resource allocation. Our results include new hardness proofs, improved approximation bounds, and efficient algorithms that may be applied to both practical domains and broader theoretical frameworks.

Keywords: Proximity Problem, Transportation Problem, Sensor Networks, Evacuation Problem, Air Quality Monitoring, Dynamic Networks, Minimum Target Coverage, Approximation Algorithm, NP-completeness, Fixed-Parameter Tractability (FPT), Min-Sum Flow Problem, Discrete and Weighted Flow Problem, Bin Packing Problem, Minsum Bin Packing, Dynamic Flow Network, Theory of Computation, log-APX-Hardness, Nearest-Neighbor Classification, Minimum Consistent Subset, Trees, Interval Graphs, APX-Hardness, Graph Algorithms, Minimum Selective Subset, Unit Interval Graphs, Polynomial-time Algorithms.