Abstract

The present thesis focuses on initial and boundary value problems for certain system of quasilinear partial differential equations (PDEs). Our objective in this thesis is to study limiting behaviour of solutions to these problems. We employ a variety of techniques such as vanishing viscosity, flux perturbation, zero relaxation limit and energy method to investigate the limiting behaviour of these system of quasilinear PDEs.

First, we study the Riemann problem for a strictly hyperbolic system of conservation laws, which occurs in gas dynamics and nonlinear elasticity. The existence and uniqueness of the solution of Riemann problem containing delta shock wave is established by employing self-similar vanishing viscosity approach. Next, we explore the limiting behaviour of solution to the Riemann problem for a macroscopic production model with van der Waals equation of state. The Riemann solution is constructed and vacuum state is observed for certain initial data. In the limiting case we establish the delta shock wave and vacuum state solution. But it is observed that the delta shock solution of the governing system is different from that of pressureless gas dynamics system so a perturbation to the flux is made and the intrinsic phenomena of concentration and cavitation is examined in the limiting case. Numerical simulations are also performed to note the effect of van der Waals parameter on the solution of the Riemann problem and to observe the formation of delta shock and vacuum state in the limiting cases.

Next, we analyze the zero relaxation limit for a Keyfitz-Kranzer type system with stiff relaxation which arises in traffic flow and nonlinear elasticity. We establish the existence of solution to the relaxation system by applying the invariant region theory. Then as the relaxation parameter tends to zero we prove that the solution of the relaxation system converges to a weak solution of the corresponding scalar conservation law. Further, we explore the asymptotic behavior of solutions of a hyperbolic system of conservation laws with linear damping on the quarter plane $(x,t) \in \mathbb{R}^+ \times \mathbb{R}^+$. The global existence of solution to the initial boundary value problem is first established under the constraints of small initial data and perturbations, which subsequently converges to their respective nonlinear diffusion wave. Also optimal convergence rates are established.

Finally, we are concerned with the inflow problem of the bipolar quantum Navier-Stokes-Poisson system in one dimension which describes the transport of charged particles in semiconductor devices with quantum effects. By exploiting center manifold theory, we first establish the existence of stationary waves with spatial decay rate provided the boundary strength is sufficiently small. Finally using energy estimates and Poincaré type inequalities, we obtain the asymptotic stability of stationary wave under smallness assumption of initial and boundary data.

Keywords: Riemann problem; Elementary waves; Delta shock wave; Vacuum state; Self-similar vanishing viscosity; Macroscopic production model; Keyfitz-Kranzer system; Relaxation limit; Compensated compactness method; Nonlinear diffusion waves; Inflow problem; Stationary solution