Abstract

This thesis titled "On the spectrum of uniform hypergraphs" explores the spectral properties of hypergraphs through the study of various matrices associated with them, including the adjacency matrix, Laplacian matrix, and signless Laplacian matrix.

First, we study the spectral properties of the Laplacian and signless Laplacian matrices of hypergraphs $\mathcal{H}$. We established a class of 3-uniform hypergraphs for which the second largest Laplacian eigenvalue is greater than or equal to the second highest degree. Meanwhile, we obtain several bounds for the spectral radius of $Q(\mathcal{H})$ in terms of hypergraph invariants such as 2-degree, weak independence number, and average degree, and provide a characterization of equality. We prove that connected bipartite graphs are the only k-uniform connected hypergraphs for which the smallest eigenvalue of the signless Laplacian is 0. We prove that for any connected k-uniform hypergraph with at least two vertices, the signless Laplacian spread is greater than or equal to $\frac{k}{k-1}$ and characterize the equality.

Next, we analyze the energy of a hypergraph $\mathcal{H}$ as the sum of the absolute values of all eigenvalues of the adjacency matrix $\mathcal{A}(\mathcal{H})$. Removing an arbitrary hyperedge from a hypergraph can affect the energy. All cases are possible: increased, decreased, or unchanged. Two distinct classes of hypergraphs are identified in which energy consistently decreases when an arbitrary hyperedge is deleted.

Finally, we extend the concept of Laplacian energy and the Kirchhoff index to hypergraphs. For the Laplacian energy, we proved that the Laplacian energy of a hypergraph is never an odd integer. We also demonstrate that removing an arbitrary hyperedge from a hypergraph can affect the Laplacian energy. Specific classes of hypergraphs are identified where energy decreases or increases upon

hyperedge deletion. Regarding the Kirchhoff index, we demonstrate that among all k-uniform hypergraphs, the k-uniform complete hypergraph achieves the minimum Kirchhoff index.

Keywords: Hypergraph, adjacency matrix, Laplacian matrix, signless Laplacian matrix, energy, Laplacian energy, Kirchhoff index.