Abstract

In this thesis, we propose to study certain K-theoretical properties of subrings of polynomial rings. More specifically, we study projective modules, symplectic modules, and their related topics, such as the K_0-stability, $\mathrm{K}_0\mathrm{Sp}$-stability, efficient generation of ideals, and the Euler class group over some class of subrings of polynomial rings.

Let A be a symbolic (or an extended symbolic) Rees algebra (need not be Noetherian) or Rees-like algebra of dimension $d+1$. We prove that $\mathrm{Vec}_r(A)$ is surjective stable at d. However, if A is a Rees-like algebra, we prove that $\mathrm{Vec}_r(A)$ is injective stable at $d+1$. As an application of injective stability, we improve the bound for the number of generators that are needed to generate a module over Rees-like algebras.

We introduce the notion of "geometric subrings" (see Definition 5.1.1), which are Noetherian subrings A of a polynomial ring $R[t]$ of dimension $d+1$ containing the ring R of dimension $d \geq 1$. We study the efficient generation of ideals and the Euler class group over geometric subrings A. We show that if $I \subset A$ is an ideal of $\mathrm{ht}(I) \geq 2$, then $\mu(I/I^2) = d+1 \Rightarrow \mu(I) = d+1$. More generally, we prove that the $(d+1)$-th Euler class group $\mathrm{E}^{d+1}(A,L)$ is trivial. As a consequence, we show that $\mathrm{Vec}_r(A)$ is surjective stable at d. Applications are given to a set-theoretic generation of an ideal in the geometric subring A and its polynomial extension $A[X]$.

An analogous problem arises with the stability theorems of Serre and Bass for symplectic modules. Let A be a geometric subring of dimension $d+1$. We prove that the symplectic A-module $(A^{2r}, \langle , \rangle)$ is cancellative for $2r \geq d$. Further, with the help of $\mathrm{Vec}_{2r}(A)$ is surjective stable at d, we observe that $\mathrm{Vec}_{2r}^{\mathrm{Sp}}(A)$ is surjective stable at $d-1$.

In this thesis, we also focus on the stabilization bound for the functors K_1 and $\mathrm{K}_1\mathrm{Sp}$ over Rees-like algebras and its Laurent polynomial extensions.

Keywords: Projective modules; Unimodular element; Cancellation; K_0-stability; Elementary transvection; Symplectic modules; Symplectic cancellation; $\mathrm{K}_0\mathrm{Sp}$-stability; Elementary symplectic transvection; Efficient generation; Euler class group; Geometric subrings; Rees algebras; Symbolic Rees algebras; Rees-like algebras