Abstract

The present thesis deals with the Bloch and the Landau constants for certain classes of functions and on Bloch-type functions. At first, we focus on the study of the Bloch and the Landau constants for meromorphic functions in $\mathbb{D}$. To this end, we consider the class of functions $\mathcal{A}(p)$ consisting of all functions f that are holomorphic in $\mathbb{D} \setminus \{p\}$ with a simple pole at $p \in (0,1)$ with $f'(0) \neq 0$. We first derive the lower bounds of the Bloch and the Landau constants for the class $\mathcal{A}(p)$. Later, we improve the lower estimates of the Bloch and the Landau constants by considering a subclass of $\mathcal{A}(p)$. Subsequently, we prove that the lower bound of the Bloch (resp. Landau) constant for the class of functions $\mathcal{A}(p)$ depends on the Bloch (resp. Landau) constant for the class of holomorphic functions in $\mathbb{D}$. Furthermore, we study the Bloch constant for some subclasses of harmonic mappings in $\mathbb{D}$ and for some subclasses of harmonic mappings in $\mathbb{D} \setminus \{p\}$ with a simple pole at $p \in (0,1)$. It is known that if f is holomorphic in the open unit disc $\mathbb{D}$ and if, for some $c > 0$, $|f(z)| \leq 1/(1-|z|^2)^c$, $z \in \mathbb{D}$, then $|f'(z)| \leq 2(c+1)/(1-|z|^2)^{c+1}$. In this thesis, we prove a meromorphic analogue of this result. Furthermore, we introduce and study the class of meromorphic Bloch-type functions that possess a nonzero simple pole in $\mathbb{D}$. In particular, we obtain bounds for the modulus of the Taylor coefficients of functions in this class. Also, we extend the notion of harmonic Bloch, harmonic Bloch-type mappings, and linearly invariant family of harmonic mappings defined in $\mathbb{D}$ to a hyperbolic domain of the complex plane. Thereafter, we establish relation of linearly invariant family of holomorphic functions with the harmonic Bloch mappings and linearly invariant family of harmonic mappings with the harmonic Bloch-type mappings that are defined on a hyperbolic domain of the complex plane. Also, we prove that bounds of the Bloch semi-norms for harmonic Bloch and harmonic Bloch-type mappings depend on the order of the linearly invariant family of holomorphic functions and on the order of the linearly invariant family of harmonic mappings, respectively.

Keywords: Bloch theorem, Bloch constant, Landau theorem, Landau constant, Bloch functions, Taylor coefficients, Distortion theorem, Growth theorem, Meromorphic functions, Harmonic mappings with simple pole, Harmonic Bloch and Bloch-type mappings, Hyperbolic domain, Hyperbolic metric, Spherical metric, Linearly invariant family, Quasiregular mappings, (K, K')-elliptic mappings.