Chapter 1

Introduction

1.1. Background

Even today, the computation of network reliability is quite cumbersome and computationally tedious, particularly when we deal with modern complex networks such as telecommunication, transportation, power systems, or computer communication networks, which requires high reliability for their economic and safe operation. Besides, in the design of such networks, the reliability has emerged as an important parameter due to the fact that failure of these networks affects its users adversely. The interest in the area of reliability evaluation is quite evident from the numerous formulations of reliability problems and the articles, which have been appearing in the literature for the past couple of decades, thereby evolving various methodologies, techniques and algorithms to tackle the evaluation and design related issues. The reasons for the proliferations of such interests and publications of articles are to create a better understanding of the theoretical nature of network reliability and design problems on variety of networks.

The network reliability can be easily computed from the reliabilities of individual components if the network has strictly a series, a parallel or a series-parallel, parallel-series configuration. However, the computation becomes quite involved if one has to deal with a network having combinations of these configurations resulting into a complex or non-series-parallel (NSP) network. Basically, the network reliability problems are NP-hard (Proven and Ball (1983, 1984), Satyanarayan and Wood (1983) Aggrawal and Barlow (1984), Aggrawal and Satyanarayan (1984), Yoo and Deo (1988)) in nature and have for long drawn the attention of researchers from all over the world in order to develop efficient algorithms for obtaining their solution. Depending on the connectivity objective of nodes, the network reliability evaluation problem can be sub-divided into three important measures to assess the performance of a system represented by a network, *viz.*, *two-terminal reliability*, *all-terminal reliability* and *k-terminal* reliability.

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Among the several approaches (such as factoring theorem, topological formula, transformation based, state enumeration etc…) appeared in Misra (1992, 1993), the Multiple Variable Inversion (MVI) based Sum of Disjoint Products (SDP) approach has been found to provide the reliability expression in a most efficient and compact manner. However, the most time consuming process in SDP based approaches is the process of obtaining disjoint of terms appearing at their input of nature, which in turn depends on the type of reliability measure to be evaluated and the number of terms to be processed. There is not only an exponential rise with number of such inputs due to increase in network size and complexity but also it may take several hours of CPU time and employ a large chunk of memory to process such information. The SDP based techniques are undoubtedly simple, do not require complex mathematics or graph-theoretical concepts and are matured enough to efficiently yield disjoint terms, but they need efficiently enumerated inputs, *i.e.*, minimal pathsets (cutsets) for *2*-terminal reliability, spanning trees (*g*-cutsets) for *g*-reliability and *k*-trees (*k*-cutsets) for *k*-terminal reliability. Clearly, if the number of inputs to SDP is small, the number of disjoint terms in SDP will decrease and consequently a compact reliability expression is obtained less amount of computational time and memory.

The present research work solely concentrates on the development of some new algorithms and techniques for processing the network data input, which can later be used for evaluating the three network reliability measures. In doing so, the work also proposes a unified framework to provide solutions to several general networks using SDP approach. Besides developing and proposing algorithms for *2*-, *g*-, *k*- terminal minimal cutsets, to extend the network reliability problem to capacitated networks, an efficient algorithm has also been proposed to enumerate irredundant subsets of minimal cutsets, which is used to evaluate the *2*-terminal reliability of capacitated networks. Additionally, it also covers some aspects of network layout design for capacitated networks.

1.2. Graph Theory: A Tool for Network Reliability Evaluation

Graph theory has drawn increased interest of scientists and engineers in the last several decades. The main reason for this accelerated interest in graph theory is in its

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demonstrated ability to solve problems from a wide variety of areas. Because of their intuitive diagrammatic representation, graphs have been found extremely useful in modeling systems arising in physical science, engineering, social sciences and economic problems. Indeed, reliability engineering has not been an exception.

The application of graph theory to reliability studies received little attention till 1970. Ever since the application of the graph theory for network reliability evaluation was suggested by Misra and Rao (1970), a large number of studies have appeared in the literature. To quote Singh and Proctor (1976): "*Until 1970, the subject received little attention with the exception of Shooman's (1968) popular text Probabilistic Reliability, published in 1968. Nevertheless, he did little more than mention the topic. However, Misra and Rao (Feb. 1970), in 1970, developed signal flow graphs- a development recognized as a significant step forward in the evaluation of network reliability*". After this a number of algorithms, techniques and approaches have appeared in the literature. In fact today, the use of graph theory has become inseparable from network reliability evaluation.

On the basis of reliability, networks/systems modeled through graphs have been classified as:

i. Undirected Networks

ii. Directed Networks

iii. Mixed Networks

For the purpose of network reliability analysis, such networks are often modeled as probabilistic graphs consisting of elements and nodes representing links and communication centers. Each element of the graph has a probability of success or failure, and has two terminals for their logical and functional interconnections. Therefore, a network graph $G= (V, E)$ consists of a set of vertices (or nodes) $|V|$ or n and a set of edges (or links) $|E|$ or e. If an edge connects two vertices i and j; j is said to be adjacent to i. The n number of nodes in the graph is assigned number 1, 2, 3…n sequentially. The e number of links of the network can be arbitrarily and sequentially assigned numbers. With this graph model, depending on the state (working or failed) of vertices (or nodes) and / or edges (or links) with specified probability, the network can be considered either working or failed with estimated probability. These graphs

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are used illustrate many of the definitions summoned up from time to time. The ARPA network has been shown in Figures 1.1, 1.2, and1.5 for undirected, directed, and mixed network cases, respectively. Here, $|V| = 5$ and $|E| = 7$ and source node'*s*' has been assigned by number '1' while the destination node 't' is represented by '5'.

1.2.1. Undirected Networks

An undirected reliability network is a connected graph in which the nodes are connected by undirected arcs. An undirected arc is an edge that has no arrow. Both ends of an undirected arc are equivalent *i.e.* there is no head or tail.

Undirected edges in the graph are employed to indicate two-way communication links between two nodes. They are represented as unordered pairs (*i, j*), where *i* and *j* represent the nodes, which are joined, by the communication link or edge. An edge is said to be incident upon two nodes if the two nodes are joined by the edge.

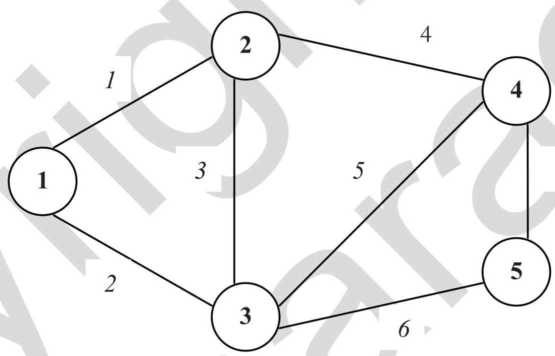

Figure 1.1: An Undirected Reliability Graph of a Network

The graph in Figure 1.1 is an example of a undirected network where,

V = {**1, 2, 3, 4, 5**}, and

E = {(**1, 2**), (**1, 3**), (**2, 1**), (**2, 3**), (**2, 4**), (**3, 1**), (**3, 2**), (**3, 4**), (**3, 5**), (**4, 2**), (**4, 3**), (**4, 5**), (**5, 3**), (**5, 4**)}

= {*1, 2, 3, 4, 5, 6, 7*}

1.2.2. Directed Networks

A directed reliability network is a connected graph, in which every branch has an orientation, *e.g.,* between the specified sources and sink nodes where source node has no branch incident to it and the sink node has no branch incident from the node.

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Directed edges are also referred to as arcs. Arcs represent one-way communication links between two nodes with communication taking place in the direction that an arc points. An arc from node *i* to node *j* is represented as an ordered pair (*i, j*), *i* is called the tail and *j* is called the head of the arc. Figure 1.2 is an example of a directed network, where,

V= {**1, 2, 3, 4, 5**}, and

E = {(**1, 2**), (**1, 3**), (**2, 3**), (**2, 4**), (**3, 4**), (**3, 5**), (**4, 5**)}

= {*1, 2, 3, 4, 5, 6, 7*}

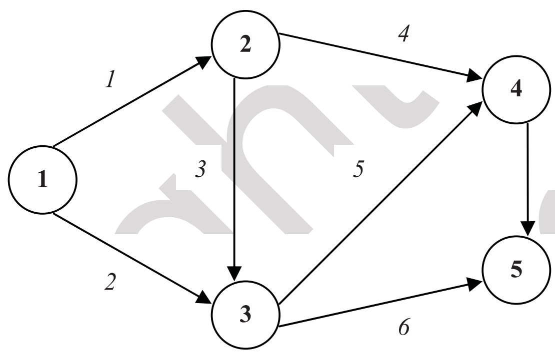
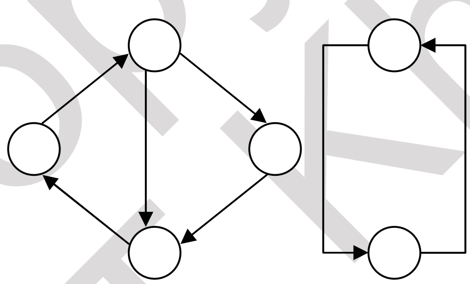
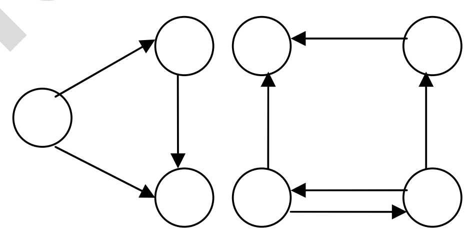


Figure 1.2: A Directed Reliability Graph of a Network



(a) (b)

Figure 1.3: Strongly Connected Components



(a) (b)

Figure 1.4: Weakly Connected Components

A directed path is an ordered sequence of nodes (1, 2, 3…*n*) in which (*i, i* + *e*) is an arc of the graph. For example (1, 2, 3, 5) is a directed path from‘*s*’ to‘*t*’. In a

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directed graph, a strongly connected component is a maximal set of nodes for which there exists a directed path between every ordered pair of nodes in the component, such that the paths pass only through nodes that are also in the component. Figure 1.3 shows two examples of strongly connected components and Figure 1.4 shows two examples of components that are not strongly connected.

1.2.3. Mixed Networks

A mixed network *G* is a graph in which some edges may be directed and some may be undirected. It is determined by the triple (*V, E, A*) where *V* is the set of nodes, *E* is the set of undirected edges and *A* is the set of directed edges. The underlying undirected graph is obtained by deleting the orientation of the arcs in *A*. An orientation of a mixed graph means, that we orient the undirected edges (and leave the directed ones).

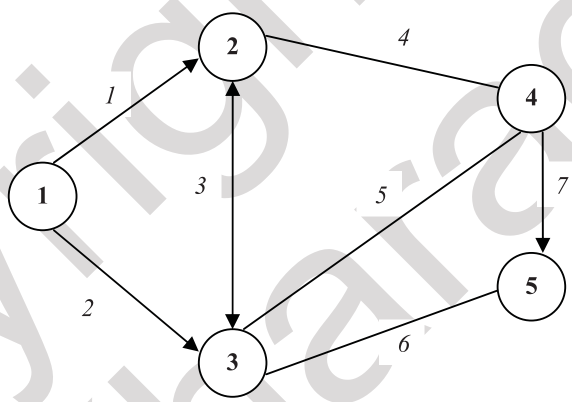

Figure 1.5: A Mixed Reliability Graph of a Network

The connection matrixes for the above cases are as shown in Figure 1.6.

Connection Matrix Figure 1.1						Connection Matrix Figure 1.2						Connection Matrix Figure 1.5					
Node	1	2	3	4	5	Node	1	2	3	4	5	Node	1	2	3	4	5
1	0	1	1	0	0	1	0	1	1	0	0	1	0	1	1	0	0
2	1	0	1	1	0	2	0	0	1	1	0	2	0	0	1	1	0
3	1	1	0	1	1	3	0	0	0	1	1	3	0	1	0	1	1
4	0	1	1	0	1	4	0	0	0	0	1	4	0	1	1	0	1
5	0	0	1	1	0	5	0	0	0	0	0	5	0	0	1	0	0

Figure 1.6: Adjacency Matrix

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