CHAPTER I

INTRODUCTION

The subject of Magnetohydrodynamics is of comparatively recent origin. As the name implies, it is the union of three widely separated disciplines, viz., Electrodynamics, Fluid Dynamics, and Thermodynamics. Naturally, MHD inherits the richness as well as the difficulties of the parent subjects.

In MHD we study the motion of an electrically conducting fluid in the presence of electric and magnetic fields. It is well known that when a conductor moves in a magnetic field, electric currents are induced in it. These currents experience a mechanical force, called the Lorentz force, due to the presence of the magnetic field. This force tends to modify the initial motion of the conductor. Moreover, the induced currents generate their own magnetic field which is added on to the primitive magnetic field. Thus, there is an interlocking between the motion of the conductor, and the electromagnetic fields.

The interlocking between the motion and the electromagnetic fields is exhibited in a more pronounced manner in liquids and gases than in solids due to the freedom of movement which the molecules of the former types of conductors enjoy. This explains why this intercoupling could not be taken seriously till the attention was fixed on liquid and gaseous conductors.

Probably the first practical application of MHD appeared when Hartmann [1] designed a magnetic pump to put mercury in motion for his experiments on the behaviour of conducting fluids in the presence of a magnetic field. The applications of MHD include direct power generation, and the control of hypervelocity vehicles. Also the concept of magnetic containment, first observed by astrophysists, is now being applied to very high temperature plasmas which are required for controlled nuclear fusion. The experimental studies of the pinched plasmas reveal that they are highly unstable; and tend to dissipate in time interval of the order of 0.5 micro second. This led to the study of instabilities in plasmas with a view to discover magnetic field geometries which will be able to stabilise these pinched plasmas over intervals, necessary for sustaining the nuclear reactions. Astrophysical phenomena produce another area for the application of MHD. The existence of the general magnetic field of the order of one gauss on the surface of the sun, the production of high magnetic fields of the order of a few thousand gauss in sun spots, the interstellar clouds which produce polarization by orienting the charged particles in the presence of magnetic fields of the order of 10^{-5} gauss, and the high energy particles in cosmic rays are some of the phenomena which have given impetus to various types of studies, for example, the investigation of the steady dynamo-mechanisms for the production of magnetic fields, the study of propagation and dissipation of hydromagnetic waves and so on.

The basic laws of electrodynamics are summarised by the Maxwell equations which define the properties of the electric and magnetic fields. These equations apply without change. However, Ohm's law, which relates the current flow to the induced voltage, has to be modified. The basic laws of hydrodynamics are summarised by the conservation equations of mass, momentum, and energy along with the thermal and caloric equations of state. The conservation equations must be modified to account for the effects of the electric and magnetic fields. The continuity equation remains unchanged. The body force, to be added to the momentum equation, is the usual ponderomotive force. Also, a derivation of the energy equation must take into account the electromagnetic theory for a moving medium; and the thermodynamics of an electrically conducting fluid [2]. If the fluid be considered non-magnetic, and the phenomena like electrostriction are neglected, the thermodynamics of an electrically conducting fluid remains essentially the same as that for a non conducting liquid. We summarise below the basic equations for an electrically conducting incompressible viscous fluid in MKS units.

The equation of continuity,

$$\nabla \cdot \vec{q} = 0, \tag{1.1}$$

The equation of momentum,

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q}\cdot\nabla)\vec{q} = \vec{X} - \frac{1}{\rho}\nabla p + \nu\nabla^2\vec{q} + \frac{1}{\rho}(\vec{J}\times\vec{B} + \rho_e\vec{E}) \quad (1.2)$$

The energy equation,

$$\frac{\partial T}{\partial t} + (\vec{q}\cdot\nabla)T = \frac{K}{\rho C_p}\nabla^2 T + \frac{\nu}{C_p}\phi + \frac{J^2}{\rho C_p \sigma} \quad (1.3)$$

Maxwell equations

$$\nabla\times\vec{B} = \mu\left(\vec{J} + \frac{\partial\vec{D}}{\partial t}\right), \quad (1.4)$$

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}, \quad (1.5)$$

$$\nabla\cdot\vec{B} = 0, \quad (1.6)$$

$$\nabla\cdot\vec{D} = \rho_e, \quad (1.7)$$

and Ohm's law,

$$\vec{J} = \sigma(\vec{E} + \vec{q}\times\vec{B}) + \rho_e\vec{q}. \quad (1.8)$$

The constitutive field equations,

$$\left.\begin{aligned} \vec{B} &= \mu \vec{H}, \\ \text{and} \\ \vec{D} &= \epsilon \vec{E}, \end{aligned}\right\} \qquad (1.9)$$

where $\vec{q}$, $\vec{B}$, $\vec{J}$, $\vec{E}$, $\vec{D}$, $\vec{H}$, $\vec{X}$ are respectively the velocity field, the magnetic induction vector, the current density, the electric field, the displacement vector, the magnetic field, the body force per unit mass in the momentum equation, and ρ, p, ν, κ, C_p, σ, μ, ρ_e, ϵ are respectively the density, the pressure, the kinematic viscosity, the thermal conductivity, the specific heat, the electrical conductivity, the magnetic permeability, the charge density, and the dielectric constant. Also ϕ is the viscous dissipation function.

Eleminating $\vec{E}$ and $\vec{J}$ from equations (1.4), (1.5) and (1.8) we obtain the equation for the magnetic induction

$$\frac{\partial \vec{B}}{\partial t} - \nabla \times (\vec{q} \times \vec{B}) - \lambda \nabla^2 \vec{B} = -\frac{\epsilon}{\sigma} \frac{\partial^2 \vec{B}}{\partial t^2} + \frac{1}{\sigma} \nabla \times (\rho_e \vec{q}), \qquad (1.10)$$

where $\lambda = \dfrac{1}{\sigma \mu}$

All physical properties of the fluid, such as ρ, ν, σ, and μ etc. are assumed to be constant. The convection current term $\rho_e \vec{q}$

the electric part $\rho_e \vec{E}$ of the body force $(\rho_e \vec{E} + \vec{J} \times \vec{B})$ per unit volume, and the displacement current $\frac{\partial \vec{D}}{\partial t}$ will be neglected in the present thesis [3]. So, equation (1.10) simplifies to

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{q} \times \vec{B}) + \lambda \nabla^2 \vec{B} .$$

Boundary conditions:

The velocity and the temperature boundary conditions remain unchanged by the addition of electromagnetic fields. The electromagnetic equations must be satisfied in the region outside the moving fluid as well. Thus, whenever the boundaries occur, or two different and immiscible conducting media are in contact with each other, there is an abrupt change in the material properties. This introduces the question of boundary conditions for electrically conducting media.

As we pass from the boundaries to the fluid, the following conditions must be satisfied: The normal component of $\vec{B}$ and the tangential component of the electric field are continuous. The jump in the normal component of electric displacement is equal to the surface charge density. The discontinuity in the tangential component of the magnetic field is equal to the current density at the surface.

If, however, both media possess finite conductivity, the tangential component of the magnetic field is continuous.

Dimensionless Parameters:

As in ordinary hydrodynamics, by transforming the basic equations to dimensionless form it can be easily proved that the following parameters govern the flow and heat transfer in magnetohydrodynamics:

(1) The Reynolds number $R = \frac{UL}{\nu} = \frac{\text{inertia force}}{\text{viscous force}}$

(2) The Prandtl number $P = \frac{\rho \nu C_p}{K} = \frac{\text{energy dissipated}}{\text{energy conducted}}$

(3) The Eckert number $E_r = \frac{U^2}{C_p(T_w - T_\infty)} = \frac{\text{Kinetic energy}}{\text{thermal energy}}$

(4) The Grashof number

$$G_r = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2} = \frac{(\text{buoyancy force}) \times \text{inertia force}}{(\text{viscous force})^2}$$

where β is the coefficient of thermal expansion; and g is the acceleration due to gravity.

(5) The magnetic-interaction parameter

$$S = \frac{\sigma B_0^2 L}{\rho U} = \frac{\text{ponderomotive force}}{\text{inertia force}}$$

(6) The magnetic Reynolds number

$$R_m = \sigma \mu U L = \frac{\text{rate of change of } \vec{B} \text{ due to fluid motion}}{\text{rate of change of } \vec{B} \text{ due to diffusion}}$$

Sometimes, the parameter $M^2 = \frac{\sigma B_o^2 L^2}{\rho \nu} = S R_m = \frac{\text{ponderomotive force}}{\text{viscous force}}$

is used instead of S . M is called the Hartmann number; and L , U , and B_o represent respectively the characteristic length, the characteristic velocity, and the characteristic magnetic field. T_w is a given temperature (say, the temperature of the solid boundary); and T_∞ is the temperature of a reference state.

The solution of an analytical problem in MHD involves the simultaneous solution of the Maxwell and hydrodynamic equations. This is undoubtedly a formidable task. However, many complicated problems can be reduced to tractable forms by neglecting some terms of the equations on the basis of some assumptions of which a few are linearization, assumption of infinite conductivity, neglect of induced magnetic field, and boundary layer approximation.

Exact Solutions:

Exact solutions in magnetohydrodynamics can be obtained only in a few cases. Hartman [1] is the first to obtain an exact solution in MHD by considering the flow between two infinite parallel plates under the action of a transverse magnetic field. Chang and Lundgren [4] extended the problem to the case of conducting walls, while Agarwal [5], and Gupta [6] included the effects of uniform suction at the plates. Yu [7], Singer [8] and Yu and Yang [9] included the effects of buoyancy force by considering the free convection flow between two vertical plates.

The Stoke's problem of flow due to an infinite plate executing harmonic oscillations in its own plane has been extended to MHD by number of workers. Ong and Nicholls [10] were the first to consider this problem. They however, neglected the induced magnetic field. Kakutani [11, 12] made a detailed and systematic study of the problem in the two cases when the plate is non-conducting and when it is perfectly conducting. The principal result of his study is that unlike the non magnetic case the flow has a double layer structure. The same problem has been considered by Axford [13].

The classical Rayleigh problem for flow due to an infinite flat plate started impulsively from rest has been extended to MHD by workers in a variety of ways. Ludford [14] was the first to study this problem. The Laplace transform technique was used; and an exact solution was obtained in the particular case when the ratio of the magnetic diffusivity and kinematic viscosity is unity. For values of this ratio other than unity, asymptotic expressions valid for large and small times were obtained. Rossow [15] reconsidered the problem. He discussed three distinct cases:

(i) when the source of magnetic field is moving with the plate

(ii) when it is fixed relative to an observer

(iii) the combination of (i) and (ii).

Nanda and Sundaram [16] pointed out that the description of case (iii) as a combination of (i) and (ii) is not correct and reconsidered this problem. Chang and Yen [17], Hasimoto [18] and Rosciszewski [19] have also analysed this problem from different view points.

The above mentioned problems may be referred to as asymptotic solutions in as much as there is only one velocity component and the resulting equations are linear. A different class of problems for which exact solutions can be obtained is the so called "Stagnation Point" flows. It is well Known that in the classical non-magnetic case the Navier-Stokes equations can be reduced to ordinary non-linear equations by suitable similarity transformations [19]. Neuringer and Mclroy [21,22] extended the two-dimensional stagnation flow to magnetohydrodynamics, while the axisymmetric case has been studied by Meyer [23], Poots and Sowerby [24] and Griben [25]. Kakutani [26] has discussed the axisymmetric case by neglecting the induced magnetic field. The two-dimensional radial flow in aconvergent or divergent channel in the presence of a circular magnetic field is another example which admits of similarity solutions. This problem has been studied by Vatagiri [27], Jungclaus [28] and Axford [29].

The magnetohydrodynamic flow due to a rotating disc presents another situation for which exact solutions of the basic equations can be obtained. Rizvi [30] looked at this problem with a weak magnetic field and assumed the disc to be a perfect conductor. However, the boundary conditions used by him are not quite correct. Sparrow and Cess [31] and Kakutani [32] studied the case of an insulating disc, while King and Lewellen [33] have analysed the inverse problem of rotating fluid over a stationary disc with an axial magnetic field. Recently Schwiderski and Lugt [34] considered in detail the problem of rotating flows over a rotating disc under very general conditions.

The magnetohydrodynamic flow between two discs are rotating and the other at rest or rotating with a different angular velocity has been studied by Srivastava and Sharma [35], Dutta [36] and Stephenson [37].

In the present thesis we have obtained exact solutions in simple closed form for two specific problems. In the Chapter III, the fully developed hydromagnetic flow in a straight rotating channel, subjected to a constant transverse magnetic field, has been analysed. An exact solution of the governing equations has been obtained; and a few particular cases of interest have been discussed. An exact solution in a simple closed form has also been obtained in the Chapter IV in the case of the magnetohydrodynamic flow between two discs, rotating with the same angular velocity Ω about two different axes, distant 'a' apart, under the action of a uniform axial magnetic field.

One of the most important problems in hydrodynamics is that of flow past a body, such as a circular cylinder or a sphere to determine the drag force experienced by the body. It is of interest to extend these problems to conducting fluids in the presence of a magnetic field. In ordinary hydrodynamics Reynolds' number controls the flow past bodies. When Reynolds' number is small, the inertia forces can be neglected; and the viscous forces are dominant in the entire velocity field, while for large Reynolds numbers the viscous forces are small compared to the inertia force everywhere in the fluid except in a thin region in the vicinity of the body where viscous and inertia forces are of comparable magnitude. Sears [38] has pointed out that there are unmistakable analogies between electrical resistance (or magnetic diffusivity) and viscosity in their effects upon flow patterns. Thus the distinguishing features of steady flows at low and high magnetic Reynolds numbers R_m are often analogous to those that differentiate steady flows at small and large Reynolds numbers R.

In magnetohydrodynamics, it is not the convection but the mechanism of Alfvén-wave propagation which, for sufficiently strong fields, carries disturbances in vorticity and current along the field lines. In flows corresponding to small R_m, however, diffusion is a dominant process; and wave-like disturbances are (excepting sound waves, if the fluid is compressible) absent altogether. The analogy between the flow patterns, corresponding to small R and R_m arises from the fact that in both situations the diffusion process is rapid compared with the convection process. In view of the large diffusion of electric current the induced magnetic field is sometimes neglected to simplify the analysis, when R_m is small.

The steady flows, corresponding to large R_m, are on the other hand, dominated by standing Alfvén waves. In the special case of aligned fields the wave character is less obvious. Since Alfvén propagation occurs along stream lines (which are similarly oriented as the field lines), the standing Alfven waves now lie along the body surface and form there boundary layer of rotational flows. When $R_m \longrightarrow \infty$, boundary layer phenomena very similar to the conventional boundary layer exist. They do, however offer a number of new features [39].

Low Reynolds Number Flow:

Chester [40] was the first to extend the Stokes problem to magnetohydrodynamics. He made the important assumption that the induced field is negligible and obtained the formula for the viscous drag D in the form:

$$D = D_s \left[1 + \frac{3}{8} M + \frac{7}{960} M^2 - \frac{43}{7680} M^3 + \cdots \right], .$$

where D_s is the classical Stokes value; and M is the Hartmann number. A different approach has been made by Ludford [41]. He takes an Oseen type approximation; and finds the drag formula

$$D = D_s \left[1 + \frac{3}{8} K \right],$$

where $K = R \quad \text{for } M^2 < R R_m,$

$$= \frac{2M^2 + R^2 - R R_m}{\sqrt{(R - R_m)^2 + 4M^2}} \quad \text{for } M^2 > R R_m .$$

Yosinobu and Kakutani [42] carried out a similar analysis for a circular cylinder. Van Blerkom [43], Imai [44], Gotah [45], and Yosinobu [46] have also analysed similar problems. A different class of problems in which the magnetic field originates within the body have been discussed by Riley [47], Murray and Chi [48], Ludford and Murray [49] and Tamada [50]

<u>Magnetohydrodynamic Boundary Layer</u>:

Rossow [51] was the first to extend Blasius problem for the steady motion of a semi-infinite flat plate to include the additional effects of an externally applied magnetic field transverse to the plate. He simplified the basic equations by the assumption that the induced magnetic field is negligible. The more important case when the applied magnetic field is parallel to the plate has been considered by a number of workers. A derivation of the boundary layer equations has been carried out by Greenspan and Carrier [52] and Davis [53,54]. In the former paper, the fundamental equations are replaced by Oseen type equations. It has been found that the boundary layer thickness continues to increase with S (the square of the ratio of the Alfven velocity to the free stream velocity), until at the critical value $S = 1$, the entire flow is brought to rest. Further discussions on the problem by means of the Oseen linearization have been given by Carrier and Greenspan [55], and Greenspan [56] respectively for unsteady flow conditions and for flow past a finite plate with $S > 1$. Glauert [57] has pointed out that the linearized method of analysis, adopted by Greenspan and Carrier cannot be relied upon in problems of such complexity. He has obtained two solutions in series valid for large and small values of the magnetic Prandtl number P_m. Glauert's treatment is, however, limited to the range $S < 1$, with $1 - S$ not small. He has further shown that boundary layer equations cease to hold when $1 - S = O(R)^{-1}$. Wilson [58] and Stewarson and Silson [59] on the other hand, have shown that solutions of boundary layer equations, derived by Greenspan and Carrier are not unique when $P_m < 1$, and $S < 1$. Other contributions include due to Jungclaus [60], Reuter and Stewartson [61], and Meksyn [62].

In all these investigations the boundary layer flow was considered in the absence of pressure gradient (i.e. uniform conditions in the free stream). The influence of the external magnetohydrodynamic pressure gradient proportional to some power of distance along the boundary layer has been analysed by Davies [63] and Griben [64]. The boundary layer flow past a magnetized plate (or cylinder) with uniform magnetic field has been discussed by Zhigulev [65], Glauert [66] and Fucks [67].

The study of free convection boundary layer in magnetohydrodynamics has received considerable attention in recent years. At first glance it appears that such flows do not have any practical application, since most heat exchangers utilize liquids whose conductivity is so small that enormously large magnetic field are necessary to influence the flow. However, some neuclear power plants use heat exchangers with liquid metal coolands so that the application of moderate magnetic fields to influence the pattern of convection appears feasible.

In the case of a hot vertical plate immersed in a quiescent fluid, heat from the surface is transferred to the fluid and causes a decrease in density so that in a field of gravity, a flow in the upward direction acts in due to buoyancy force. When a magnetic field is applied normal to the plate, the Lorentz forces tend to retard this flow, and cause a decrease in heat transfer rate to the fluid.

Lykoudis [68] has discussed similarity solution for steady MHD free convection past an isothermal vertical plate with a transverse magnetic field for a range of Prandtl numbers. Reeves [69] has examined the class of similarity solutions with variable magnetic field strengths and wall temperature distribution. Cramer [70] has shown that similarity solution is also possible for a constant magnetic field when the excess of the wall temperature over the ambient value varies directly as the distance along the plate measured from the bottom edge. He has also discussed steady fully developed flow in a parallel plate channel with linearly increasing wall temperatures in the presence of a magnetic field. Gupta [71] has given an approximate solution, by Karman-Pohlhausen method, of the boundary layer equations for free convection from a vertical plate. Sparrow and Cess [72] have studied non-similar solutions for steady MHD free convection past a semi-infinite vertical hot plate in the presence of a uniform transverse magnetic field while, Mabuchi [73] has discussed non-similar solutions with variable magnetic fields. Perhaps, the most penetrating analysis of the general flow properties in the physical situation under review, has been presented by Singh and Cowling [74] and Riley [75]. Singh and Cowling have shown that, regardless of the strength of the applied field, there will always be a region in the vicinity of the leading edge where electromagnetic forces are not important, while at large distances from the leading edge these magnetic forces dominate. Similarity solutions are possible in each of these two regions. Their solution, however, suffers from a drawback in that it ignores the presence of a very thin layer of liquid, adjacent to the wall in which viscous forces are important. Consequently the no-slip condition at the wall is violated and hence no estimate of the skin friction can be made. Their solution has been improved by Riley who has given series as well as Pohlhausen type solution, taking account of the inner (viscous) boundary layer. The study of magnetohydrodynamic free convection from a horizontal plate was initiated by Nanda [76] who obtained similarity solutions for the problem. Subsequently, Gupta [77], and Singh and Cremers [78] have given approximate solutions using the momentum integral method.

In the present thesis, we have studied in the Chapter II the free-convection boundary layer flows for high and low Prandtl numbers past a semi infinite vertical flat plate in the presence of a magnetic field, applied perpendicularly to the plate.

One important distinction between free and forced convection flows in MHD boundary layer problems may be noted. In forced convection flows, induced electromagnetic forces may modify the inviscid free stream which in turn may modify the external pressure gradient on the free stream velocity which is imposed on the boundary layer. Hence, for a complete solution in this case, one should solve the inviscid free stream problem for determining the boundary layer characteristics. On the other hand in free convection flows, the velocity being zero outside the boundary layer, the induced electromagnetic forces are absent there. Thus the influence of the magnetic field on the boundary layer is exerted through the electromagnetic forces confined to the boundary layer only, with no additional effects arising out of the free stream pressure gradient.

Hall effect

It has been observed that in an ionized gas where the density is low, and/or the magnetic field is strong so that the cyclotron frequency $\omega = eB_0/m$ exceeds the collision frequency (where e and m are the charge and mass of an electron) the electron can make a number of cyclotron orbits between two successive collisions. This causes it to drift in a direction perpendicular to the direction of the magnetic and electric fields. Thus if an electric field be applied at right angle to the magnetic field, the whole current will not flow along the electric field. This tendency of the electric current to flow across an electric field in the presence of a magnetic field is called Hall effect. This effect is likely to be important in many astrophysical situations as well as in flows of laboratory plasma. The generalized Ohm's law taking into account the contribution to current due to Hall effect, is

$$\vec{J} = \sigma\left(\vec{E} + \vec{q}\times\vec{B} - \frac{1}{ne}\vec{J}\times\vec{B} - \frac{1}{ne}\nabla p_e\right),$$

where the electrical conductivity σ is defined in the absence of the magnetic field, n is the number density of electrons, and p_e is the electron pressure; and τ is the mean time between successive collisions.

Hall effects in steady flows of a partially ionised gas between two stationary parallel plates have been studied by Sato [79], Kusukawa [80] and Sherman and Sutton [81]. On the other hand, the Couette motion of an ionised gas between two parallel plates with Hall effects, has been investigated by Chekmerev [82]. The heat transfer characteristics of these flows have been considered by Gupta and Chatterjee [83]. Naruse [84,85,86,87] in a series of papers, has considered the boundary layer flows past an electrically insulating body, the applied magnetic field being parallel to the free stream. In a subsequent paper [88], he has also studied the case of transverse applied magnetic field. As a particular case he considered the flow past a semi-infinite plate and obtained similarity solutions. Katagiri [89] has recently studied non-similar solutions using finite differences.

In the Chapter VI of this thesis, the effect of the Hall currents on two specific flow problems in the presence of a uniform transverse magnetic field has been studied. In the first part of the chapter, the hydromagnetic Rayleigh problem has been discussed. An exact solution of the problem has been obtained with the help of Laplace transform technique. Limiting cases for small and large times have been discussed.

In the second part, we have considered the effect of Hall currents on the flow due to a disc, performing torsional oscillations about a state of rotation with constant angular velocity Ω in a fluid which is also rotating with the same angular velocity.

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