SYNOPSIS

The thesis deals, with the formulation and application of variational principles to the problems of free and forced convective heat transfer in incompressible viscous fluids. In the introduction, the application of variational techniques in the field of heat transfer is briefly reviewed.

The thesis is divided into five chapters.

Chapter I

VARIATIONAL PRINCIPLES FOR THE UNSTEADY HEAT CONVECTION

A variational principle for unsteady forced heat convection in laminar flow is formulated. It is based on the thermodynamics of linear irreversible processes. The phenomenon of heat convection is represented by a heat flow vector field $\mathbf{H}$ instead of a scalar temperature field θ. The concepts of thermal potential V, dissipation function D and generalized thermal force are introduced. The variational principle equivalent to the equation of heat convection and leading to the equations of Lagrangian type, is stated as, $\delta V + \delta D = \iint_S \theta \, \mathbf{n} \cdot \delta \mathbf{H}' \, ds$; the surface integral extends over the boundary S of the domain with the unit normal $\mathbf{n}$ to the boundary taken positive inward. The effects of surface heat transfer and nonuniform surface temperature are also included. The principle is extended to include systems where dependence of heat capacity and conductivity on temperature leads to a nonlinear energy equation.

Chapter II

VARIATIONAL PRINCIPLES FOR THE STEADY HEAT CONVECTION IN DUCTS

A variational principle is developed for the problems of heat transfer in channel flows for large Péclét numbers (axial heat conduction neglected). The equation of <u>steady</u> heat convection in three dimensions is transformed into a two-dimensional <u>unsteady</u> equation by introducing a time-like variable. Thermal potential and dissipation function are defined as in Chapter I and a similar variational principle is formulated. The effects of surface heat transfer, nonuniform temperature on the periphery of the duct and of the temperature dependent parameters are included.

Chapter III

APPLICATION OF THE VARIATIONAL PRINCIPLE TO THE PROBLEMS OF STEADY HEAT TRANSFER IN DUCTS

The variational principle formulated in Chapter II, is applied to (1) Heat transfer between parallel plates for (i) slug flow, (ii) parabolic flow and (2) Heat transfer in a tube of circular section for (i) slug flow, (ii) parabolic flow. Comparison of the results with the available exact solutions is made and the agreement is found to be good. A method is developed to estimate the Nusselt number at infinity without detailed calculations for the temperature distribution.

Chapter IV

A VARIATIONAL METHOD FOR COMBINED FREE AND FORCED CONVECTION IN CHANNELS

A variational method is presented for heat transfer problems of combined free and forced convection by a fully developed laminar flow in vertical channels. The momentum and energy equations are combined to give an inhomogeneous Helmholtz wave equation with homogeneous Dirichlet boundary conditions. This equation is solved variationally for the particular case of flow in a rectangular channel.

Chapter V

HEAT TRANSFER IN LAMINAR FLOW BETWEEN PARALLEL PLATES AT SMALL PÉCLÉT NUMBERS

The problem of heat transfer between two infinite parallel plates is studied to consider the effect of heat diffusion on the incident fluid. The velocity distribution is taken to be parabolic. The heat generated by viscous dissipation is also taken into account. Solutions of the energy equation for the regions $x \leqslant 0$ and $x \geqslant 0$ are obtained and the continuity conditions are imposed on the temperature and its derivative at the junction $x = 0$. Detailed solutions are given for $Pe' = 1$. Mean mixed temperatures and Nusselt numbers are calculated. It is found that at small Péclét numbers the incident temperature is affected by the diffusion of heat from right $(x > 0)$ to the left $(x < 0)$ of the section $x = 0$. The Nusselt number at infinity for the parabolic velocity distribution is smaller than that for the slug flow.