SYNOPSIS

The thesis contains the results of theoretical investigations on Cherenkov effect and electrodynamics of continuous media. The motivation of our present study was a desire to set up a theory of Cherenkov effect more in tune with the principle of relativity and sufficiently powerful to enable a solution of the problem of radiation in biaxial crystals. The consideration of the covariant features of the theory, however, led to some results in the electrodynamics of continuous media itself. All the discussions in this thesis are based on the classical theory.

The thesis comprises five chapters. The accounts of our investigations are prefaced by an introductory chapter devoted to the review of the general aspects of Cherenkov effect. Efforts have been made to discuss the practical implications of the effect in some detail. Due to the author's lack of knowledge of Russian and the unavailability of many original papers no attempt could be made towards comprehensiveness of the review.

In Chapter II we discuss the general electrodynamics of uniformly moving media. It is suggested that the well known difficulty associated with the unphysical nature of the usual field tensors can be avoided by adopting the Minkowski four forces f_λ f_λ^* as the field variables. By utilising a projection technique outlined by Weyl we write the Maxwell equations in terms of these variables. The resulting equations resemble the three dimensional form of Maxwell equations. The relation between the two alternative forms of the constitutive equations is also discussed. We then elaborate upon the symmetry of the basic electrodynamic equations with respect to the electric and magnetic quantities. For some cases, this symmetry enables one to obtain the magnetic field from electric field and vice versa. It turns out that the two independent modes of wave propagation in an anisotropic medium are related through this symmetry. We next set out to solve the new form of Maxwell equations. It is found that for this set it is no longer necessary to introduce the potentials and as a consequence the imposition of a gauge condition is not called for. Apart from this methodological advantage the solutions obtained are of greater generality inasmuch as they include the effects of magnetic monopoles and their currents as well.

Chapter III of the thesis is devoted to a discussion of Cherenkov effect in uniaxial media. We solve the Maxwell equations in the proper frame of the charge in which the field is static. Instead of introducing the usual four-potentials we describe the field by two scalar potentials defined by $\bar{E} = \text{grad}\, \mathcal{G}$, $\bar{H} = -\text{grad}\, \Upsilon$ After elimination of $\bar{B}$ from the field equations we are led to a fourth order partial differential equation for $\mathcal{G}$. It is shown that the fourth order operator factorises into two second order operators under conditions somewhat more general than the usual restrictions of uniaxiality. For non-dispersive media the two operators are simultaneously diagonalised by transformations having properties similar to Lorentz transformations. The field of an electric charge is then found by Fourier analysis. The field becomes infinite on a quartic cone which factorises into a pair of quadric cones and vanishes beyond it. The reciprocal of this is the Cherenkov cone of wave normals for which a neat expression is also given. In the calculation of the total energy of radiation it is essential to consider effects of dispersion and another set of transformations become necessary for diagonalisation of the operators.

The theory of Cherenkov effect in biaxial media is discussed in Chapter IV. For the solution of this problem we adopt a dual approach. $\bar{B}$, $\bar{D}$ and $\bar{H}$ are eliminated from the field equations in the rest frame of the medium by utilising the powerful methods of tensor algebra. The field in the proper frame of the charge is then found by making a special Lorentz transformation. The integration of the Fourier integrals for the fields is greatly facilitated by the recognition of a simple connection between the denominator of the Fourier integrals and Fresnel's wave surface. We discuss a generalised form of the wave surface for media anisotropic in both dielectric and magnetic permeabilities for which the principal axes are not necessarily parallel. We show that in the case of combined parallel anisotropy a few transformations reduce the generalised surface to the Fresnel form. An immediate consequence of this result is that for motion along a principal axis the substance can be assumed to be magnetically isotropic; the radiation loss for combined isotropy being obtainable from the dielectric case by means of these transformations alone. The actual integration of the Fourier integrals is done for the special case of motion of the charge along a principal axis by utilising the parametrisation of the wave surface given by Weber. We also give an alternative derivation of Weber's result. The resulting elliptic integrals are calculated by standard methods leading to the expression of the total energy in terms of Heuman's Lambda function and complete elliptic integral of the first kind. The final expression assumes different forms in the five distinct cases which arise depending on the relative magnitudes of the principal dielectric constants and the particle velocity. In each case, however, the sum of the energies of the two modes takes a form similar to the result of Frank and Tamm.

The final Chapter is devoted to the considerations of energy and momentum of the electromagnetic field. We propose a new definition of the energy flux four vector for free electromagnetic fields in material media. It is shown that for this definition the old concept of duality between the wave and ray vectors of a plane electromagnetic wave can be extended. This duality involves a substitution scheme which is different from the symmetry between the electric and magnetic variables. We generalise the Fresnel wave surface to media in motion. The dual of this is the ray surface which gives the locus of the electromagnetic disturbances spreading out from a momentary event at $x_i = 0$. The relation between these surfaces and the Cherenkov and ray cones is elaborated. For some simple physical situations we discuss the balance of electromagnetic energy and momentum from the standpoint of the two energy momentum tensors of Minkowski and Abraham. We end our discussions by deducing some formal properties of the energy momentum tensors.

The principal results given in Chapters II and V have been communicated for publication in the form of two papers.

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