CHAPTER I

INTRODUCTION

1.1. FLOWS DUE TO ROTATING DISKS IN ORDINARY VISCOUS FLUIDS

In 1921 von Kármán initiated the study of the flows associated with the rotating disks. He considered the flow engendered by a rotating disk in a fluid otherwise at rest and found that the rotating disk behaves like a centrifugal fan throwing the fluid near its surface radially outwards and this motion in turn induces an axial flow towards the disk. He obtained expressions for the velocity components and the frictional torque which have been later experimentally verified both in laminar and turbulent conditions by Theodorsen and Regier (1944), Smith (1947) and Gregory and Walker (see Gregory et al. 1955). The numerical solutions for the flow were given by Cochran (1934), Jain (1960, 1962a, 1962b) and Benton (1966) in the laminar case and by Goldstein (1935) in the turbulent case. Bödewadt (1940) studied a related problem in which the disk is at rest and the fluid at infinity is rotating. Recently, Schwiderski and Lugt (1964a) gave an analytical solution to both the flows of von Kármán and Bödewadt. They assumed a form of the boundary-layer thickness distribution which is a function of radius and which allows another boundary condition at large radii from the axis of rotation. With the help of a boundary-layer transformation, they reduced the Navier-Stokes equations to a set of ordinary differential equations. They transformed this set to a set of Volterra integral equations and solved them by means of an iteration procedure.

In 1953 Nigam discussed the rotation of an infinite plane lamina in a compressible fluid. He assumed the flow to be isentropic and integrated the equations for a particular case in which the fluid obeys the pressure-density relation, $p = p_0 + K\rho^{5/3}$. He found that the essential features of the flow are the same as in the corresponding problem of von Kármán (1921) for an incompressible fluid.

The flow due to a rotating disk with suction was studied theoretically by Stuart (1954) and experimentally by Gregory and Walker (1960). A similar flow in which the disk is naturally permeable was recently treated by Joseph (1965). Nanda (1961) discussed the influence of suction on the revolving flow of a viscous incompressible fluid over a fixed flat plate. He found that when the suction velocity increases, the displacement and momentum thicknesses corresponding to the tangential component of flow decrease.

The steady forced flow of a viscous incompressible fluid against a rotating disk was studied by Hannah (1947), Schlichting and Truckenbrodt (1952) and Tifford and Chu (1952) in the laminar case and by Truckenbrodt (1954) in the turbulent case. Wadhwa (1963) considered the unsteady stagnation flow towards a rotating plane lamina. He obtained the solution when the rotation of the lamina and also the stagnation flow at infinity are started either impulsively or with a uniform acceleration.

Thiriot (1940,1950) studied the boundary-layer flow shortly after a sudden start or stop of a rotating disk. Nigam (1951) computed the details of the boundary layer growth on a disk started impulsively from rest. Wadhwa (1958) discussed the unsteady rotation of a plane lamina in accelerated motion. Recently, Benton (1966) gave an exact analysis of the unsteady flow due to an impulsively started disk. Muhuri (1966) investigated the problem of a disk rotating steadily about its axis when its angular velocity is impulsively altered by a small quantity $\epsilon\Omega$. He assumed that the velocity components differ from the solution of von Kármán (1921) by a term proportional to ϵ , and linearized the unsteady equations in this parameter. He discussed the resulting linear equations for small and large times. For large time he used a Pohlhausen technique, while for small time he showed that the first terms are analogous to the solution of Rayleigh (1911) for the impulsive motion of a flat plate.

In 1959 Kanwal studied the problem of impulsive rotatory motion of a thin circular disk in a viscous fluid. Later Majumdar (1962a) solved the same problem by assuming that the impulsive shearing stress on the disk varies linearly as the distance from its centre. In a note on this paper (1962b), he discussed another important case for which the impulsive shearing stress is given by

$$\left.\begin{aligned}(p_{\theta z})_{z=0} &= p_0 (a^2 - r^2)^{1/2}\, \delta(t) \qquad & r<a \\ &= 0 & r>a\end{aligned}\right\},$$

where p_0 is a constant and $\delta(t)$ is the unsymmetric Dirac delta function the Laplace transform of which is unity.

Sparrow and Gregg (1960) solved the unsteady flow about a rotating disk with varying angular speed by computing the deviations from the quasi-steady state. In a comment on this flow, Lu (1960) obtained differential equations for the case in which the angular velocity Ω of the disk is expressible as $\Omega = a_1 (t + a_2)^{-1}$, $a_2 > 0$. Dobryšman and Sadokov (1960) studied another unsteady flow in which the disk rotates with an angular velocity $\Omega(t)\, H(t)$, where $H(t)$ is a unit step function of time t . They obtained the solution by assuming that $\Omega(t)$ is expressible in a power series of the form

$$\Omega(t) = \sum_{n=2}^{\infty} \Omega_n \left(\sqrt{t}\right)^n H(t).$$

Rosenblat (1959), Benney (1964) and Riley (1965) investigated the torsional oscillations of an infinite plate in a viscous incompressible fluid. The same flow with large suction was discussed by Kelly (1965). Mamaladze and Matinian (1960) solved the problem of torsional oscillations of a disk about its axis in a rotating fluid. Mamaladze (1964) studied the deeply penetrating transverse waves induced by a disk oscillating in a rotating fluid. He showed that as the frequency of rotation of a fluid approaches the half frequency of the axial oscillation of a disk immersed in it there is an abrupt increase in the depth of penetration of viscous waves which are excited by the oscillations of the disk. Recently, Benney (1965) considered the flow of a viscous incompressible fluid when an infinite disk performs torsional oscillations about a steady mean and the fluid at infinity rotates with a constant angular velocity. The same flow when the fluid at infinity is at rest was considered by Sharma (1964) and Rarity (1965).

The flow due to rotating disk with a rotating fluid at infinity was studied by Batchelor (1951), Stewartson (1953), Rogers and Lance (1960) and Schwiderski and Lugt (1964b,1965a). The works of Batchelor and Stewartson were critically examined by Moore (1956) in a review article. The case in which both the disk and the fluid at infinity are rotating in the same sense was approximately treated by Fettis (1955) by means of an iteration procedure. His results show that when the fluid at infinity rotates faster than the disk, the velocity profiles are of the Bödewadt type, and that even when the fluid at infinity rotates slower than the disk, the transverse velocity distributions have a slight overshoot.

The steady flow of an incompressible viscous fluid between two concentric infinite rotating disks was investigated by Batchelor (1951), Stewartson (1953), Grohne (1955), Lance and Rogers (1960,1962) and Kolokol'tsov (1963). The unsteady flow between two torsionally oscillating disks was discussed by Rosenblat (1960). The same flow with one of the disks stationary and porous was recently treated by Jawa (1966). The effect of time-dependent injection on the unsteady flow between two infinite rotating disks was studied by Sharikadzé (1963). The time-dependent flow between two impulsively started disks was solved by Pearson (1965).

Schultz-Grunow (1935) and Soo (1958) studied the flow over a disk rotating in an enclosure. Stewartson (1957a) solved the problem of the boundary layer flow in the neighbourhood of the edge of a disk in a rotating flow. Recently, Rogers and Lance (1964) numerically examined the boundary layer on a stationary disk of finite radius in a rotating flow. The same problem when the external flow varies in a power-law manner was previously treated by Mack (1962) and King (1964) using the momentum-integral method.

In a recent investigation, Nigam and Subba Rao (1965) pointed out that the similarity solutions given by von Kármán (1921) for the flow due to a rotating disk not only correspond to a separation of variables in axisymmetric flow, but they are also motivated by group-theoretic considerations.

1.2. ROTATING DISK FLOW PROBLEMS IN MAGNETOHYDRODYNAMICS

In recent years, much work was done on the generalization of well known viscous flow solutions to take account of the additional effects of a magnetic field when the fluid is electrically conducting. An important class of such solutions is the rotating hydromagnetic flows of incompressible viscous fluids which are of great importance in magnetohydrodynamic generators. When an electrically conducting fluid flows in the presence of a magnetic field, electric currents induced in the fluid modify the field and produce mechanical forces which modify the motion. The development of the entire subject of Magnetohydrodynamics+ is based on this interaction between a moving electrically conducting fluid and an externally applied magnetic field. In the following paragraphs,

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+ Exhaustive treatment of the subject is made in the text books by Cowling (1957), Ferraro and Plumpton (1961), Pai (1962), Alfven and Fälthammar (1963), Cambel (1963), Bhatnagar (1964), Shercliff (1965), Alan Jeffrey (1966) and Hughes and Young (1966).

we shall briefly survey the works of various authors on the flows about rotating disks in magnetohydrodynamics.

The problem of hydromagnetic flow due to a finite rotating disk was studied by Stewartson (1957b). This flow gives a theoretical explanation to some of the experiments of Lehnert (1955) in magnetohydrodynamics. Stewartson considered the effect of a vertical magnetic field on the motion of a shallow dish of mercury part of whose base is rotating while the rest is fixed. He found that both in the absence of magnetic field and when the field is large there is a steady solution in which the mercury is either uniformly rotating or at rest, there being a thin friction layer separating the two regions. In obtaining the solution, Stewartson neglected the radial component of the magnetic field in comparision with the strong external field imposed along the axis of rotation. In a note on the above problem, Majumdar (1958) proved that for a rotationally symmetric steady hydromagnetic field the radial component of magnetic field can always be neglected in comparision with the field along the axis of rotation if the depth of the liquid column is small compared with the radius of the disk.

In 1960 Ray discussed the influence of a uniform magnetic field on the couple resisting the rotation of a circular disk in a viscous conducting fluid. He found that, when the field is weak, the field helps the rotation on one side and retards it on the other. Sharma (1963) studied the effect of magnetic field on the flow over an enclosed rotating disk. He found that the moment on the rotating disk increases with an increase in the Hartmann number.

The hydromagnetic flow over a rotating disk of infinite dimensions was investigated by Sychev (1960), Sparrow and Cess (1962), Kakutani (1962), Rizvi (1963) and Datta (1966). Sparrow and Cess (1962) and Kakutani (1962) neglected the induced magnetic fields while Sychev (1960), Rizvi (1963) and Datta (1966) attempted to solve the problem with full magnetic field. This problem belongs to the class of exact solutions of the magnetohydrodynamic equations described by Lin (1958). Jagadeesan (1964) studied the hydromagnetic stagnation flow against a rotating infinite disk with the help of Kármán-Pohlhausen method. He found that the torque on the disk increases as the strength of the applied magnetic field increases. Pande (1964) and Suryaprakasa Rao and Gupta (1966) discussed the hydromagnetic flow against a rotating disk with large suction.

Srivastava and Sharma (1961) considered the motion of a conducting fluid between two infinite disks when one of them is rotating with a constant angular velocity and the other is at rest. They assumed a constant magnetic field to act normal to both the disks. The solution obtained by them is valid for small hydrodynamic Reynolds number since only the first two terms of a series expansion in Reynolds number are considered. They found that when the magnetic field strength increases, both the shear stress and the force of suction decrease on the stationary disk. Recently, Datta (1966) studied the flow between two infinite slowly rotating disks with full magnetic field. He obtained the solution by expanding the velocity components and magnetic field in ascending powers of the Reynolds number. He found that when the disks rotate in the same sense with equal angular velocities, there is no interaction between the flow and magnetic field. Nihoul (1964) discussed the slow motion of a conducting fluid between two rotating plane walls. He assumed that the flow is under the influence of a magnetic field set up by a line-current along the axis of rotation and obtained expressions for the velocity and pressure gradient for low hydrodynamic and magnetic Reynolds numbers.

S.Datta (1964), Kelly (1964) and N.Datta (1964) studied the effect of a uniform magnetic field on the torsional oscillations of an infinite plate in a viscous conducting fluid. The analogous problem in which the fluid is bounded by another oscillating disk was investigated by Bhatnagar (1961), Devanathan (1962) and S.Datta (1965). Recently, Gupta (1966) studied the transverse waves induced by a disk oscillating about a state of steady rotation in magnetohydrodynamics. Contrary to the non-conducting case (see Benney 1965), he found that in the presence of magnetic field, none of the Ekman-Stokes layers becomes infinite when the angular velocity of rotation of the fluid approaches the half frequency of oscillation of the disk.

In 1964 Lewellen and King investigated the boundary layer of a conducting vortex flow over a stationary (insulated) disk with an axial magnetic field. In a subsequent paper, King and Lewellen (1964) generalized the similarity solution of Bödewadt (1940) to consider a power-law variation in the external flow and to consider the case of the boundary layer produced by the rotating flow of a conducting fluid interacting with an axial magnetic field. Schwiderski and Lugt (1965b) studied the influence of electromagnetic forces on the rotating flows of electrically conducting fluids over a rotating infinite disk with the help of adjustable local boundary layer approximations.

1.3. FLOW PROBLEMS DUE TO ROTATING DISKS IN NON-NEWTONIAN FLUIDS

We know that a Newtonian fluid is characterized by a linear relationship of the form

$$p'_{ik} = 2\mu e_{ik}, \tag{1.3.1}$$

where $p'_{ik} (= p\delta_{ik} + p_{ik})$ is the deviatoric stress tensor, $e_{ik} \left[= \frac{1}{2}(v_{k,i} + v_{i,k}) \right]$ the rate-of-strain tensor and μ the coefficient of viscosity. The theory based on the equation (1.3.1) could successfully explain the phenomena of skin-friction, lift, form drag, separation, etc. However, it failed to explain many interesting phenomena like Merrington effect, Weissenberg effect and Reiner effect. Merrington (1943) observed that when a solution of rubber in mineral oil is forced through a straight pipe, the fluid swells on emerging from the pipe. As pointed out by Merrington, this phenomenon of swelling is due to the elastic recovery of the fluid stretched in the pipe. Weissenberg (1947) found that when a high polymer solution is sheared between two coaxial cylinders (keeping the inner cylinder at rest and the outer cylinder in rotation), the liquid climbs up the inner cylinder against the action of centrifugal force. This phenomenon of climbing of the liquid in a direction perpendicular to the plane of shearing is known as Weissenberg effect or Normal stress effect. In 1957 Reiner noticed that when air is sheared in a small gap between two coaxial disks (one rotating and the other at rest), the non-rotating disk experiences a thrust at its centre. The fluids exhibiting these effects include many other industrial products like plastics, synthetic fibers and slurries. All fluids which do not obey the equation

(1.3.1) are referred to as Non-Newtonian fluids+. During the last two decades, many mathematical models are developed to explain the flow behaviour of these fluids. We give below a brief description of some of these models in which the problems of rotating disks are studied.

(i) Reiner-Rivlin Fluids

This class of fluids is governed by the constitutive equation

$$p_{ik} = -p\delta_{ik} + 2\mu e_{ik} + 4\mu_c e_{ij} e_{jk} \,, \qquad (1.3.2)$$

where p_{ik} is the stress tensor, e_{ik} the rate-of-strain tensor, p the isotropic pressure, δ_{ik} the kronecker delta, and μ and μ_c are the viscosity and cross-viscosity coefficients which are, in general, functions of the three invariants of rate-of-strain tensor besides being the functions of the material properties. The equation (1.3.2) was proposed by both Reiner (1945) and Rivlin (1948) and therefore the fluids governed by it are known as Reiner-Rivlin fluids. The introduction of the cross-viscosity term in the constitutive equation is responsible for attributing to the fluid the Weissenberg and Merrington effects.

+ A detailed account of these fluids can be found in the works of Metzner (1961), Wilkinson (1960), Bhatnagar (1962), Kapur (1962) and Fredericksen (1964).

During the last decade, a large amount of work was done in the rotating-disk flows of Reiner-Rivlin fluids. Srivastava studied the rotation of an infinite plane lamina (1958), the rotatory oscillation of an infinite plate (1959, 1960) and the flow at small Reynolds number between two infinite disks, one of which is rotating and the other is at rest (1961). Nigam (1958) discussed the impulsive rotation of an infinite plane lamina in Reiner-Rivlin fluids. He observed that when an infinite plane lamina rotates impulsively from rest in a Reiner-Rivlin fluid, the axial flow is directed away from the lamina and the fluid moves radially inwards near it, a phenomenon contrary to the ordinary viscous case. Mithal (1958,1961) investigated the motion produced by the uniform rotation of an infinite plate with or without suction. Jain (1961,1962c, 1964) studied the flow about a rotating disk with or without a forced flow from infinity towards the disk. He and Balaram (1961) discussed the effects of uniform high suction on the flow against a rotating disk and on the revolving flow over a fixed flat plate. Balaram (1963, 1966) independently solved the flow between two infinite slowly rotating disks and the hydromagnetic forced flow against a rotating disk. He and Sastri (1965) investigated the rotationally symmetric flow of a Reiner-Rivlin fluid in the presence of an infinite rotating disk. Rajeswari (1961) studied the flow between torsionally oscillating infinite disks. Bhatnagar (1963) investigated the flow between two infinite disks (one rotating and the other at rest) for large values of the Reynolds number. Rintel (1964) discussed the flow between two finite disks (one rotating and the other at rest) for small values of the Reynolds number. Lugt and Schwiderski (1966) studied the axisymmetric rotating flows of Reiner-Rivlin fluids over a rotating disk with the help of adjustable local boundary layer approximations.

Although the equation (1.3.2) explains most of the non-Newtonian behaviours, it gives in simple shearing flow that the normal stresses in and perpendicular to the plane of shear are equal. However, in most of the real fluids, we find that the difference of the two normal stresses is a function of shear rate. Hence there is no real fluid which is governed by the equation (1.3.2).

(ii) <u>Rivlin-Ericksen fluids</u>

In 1955 Rivlin and Ericksen considered a visco-elastic fluid which is isotropic when stationary and for which the stress components p_{ik} at any instant are expressible as polynomials in the gradients of velocity, acceleration, second acceleration, ... , $(n-1)$th acceleration. They showed that the stress matrix $P(=\|p_{ik}\|)$ can be expressed as a matrix polynomial in n kinematic matrices $A_1, A_2, \ldots, A_n$, defined in terms of the velocity components v_i by

$$A_1 = \| A_{ik}^{(1)} \| = \| v_{i,k} + v_{k,i} \| , \qquad (1.3.3)$$

and

$$A_{r+1} = \| A_{ik}^{(r+1)} \| = \Big\| \frac{\partial A_{ik}^{(r)}}{\partial t} + v_j A_{ik,j}^{(r)} + A_{ij}^{(r)} v_{j,k} + A_{kj}^{(r)} v_{j,i} \Big\| \; (r = 1, 2, \ldots, n-1) . \qquad (1.3.4)$$

The coefficients in this matrix polynomial are polynomial invariants of $A_1, A_2, \ldots, A_n$.

Rivlin (1955) pointed out that if the stress matrix $P (= \| p_{ik} \|)$ can be expressed as a matrix polynomial in two kinematic matrices, say $\| A_{ik} \| = A$ and $\| B_{ik} \| = B$, then P can be expressed by a relation of the form

$$P = \| p_{ik} \| = \mu_0 I + \mu_1 A + \mu_2 B + \mu_3 A^2 + \mu_4 B^2 + \mu_5 (AB + BA) + \mu_6 (A^2 B + BA^2) + \mu_7 (AB^2 + B^2 A) + \mu_8 (A^2 B^2 + B^2 A^2) , \qquad (1.3.5)$$

where I is the unit matrix and $\mu_q (q = 0, 1, 2, \ldots, 8)$ are polynomials in the traces of the ten matrices $A, B, A^2, B^2, A^3, B^3, AB, A^2 B, AB^2$ and $A^2 B^2$. If the fluid is incompressible, then μ_0 can be replaced by an arbitrary quantity $-p$.

The fluids governed by the equation (1.3.5) are called <u>Rivlin-Ericksen fluids</u>. All the rotating disk problems which were studied in these fluids are restricted to the class for which μ_0, μ_1, μ_2 and μ_3 are constants and μ_q ($q = 4, 5, \ldots, 8$) are zeros. For this class of fluids, the equation (1.3.5) is of the form

$$p = -pI + \mu_1 A + \mu_2 B + \mu_3 A^2, \qquad (1.3.6)$$

where μ_1, μ_2 and μ_3 are respectively the coefficients of viscosity, visco-elasticity and cross-viscosity.

Using the equation (1.3.6), Bhatnagar and Rajeswari (1962) investigated the secondary flows induced in a non-Newtonian fluid between two parallel infinite oscillating planes. They observed that the breakdown of the steady component of the secondary flow at critical values of the cross-viscosity and visco-elastic parameters, followed by the reversal of flow at higher values, is a characteristic feature of non-Newtonian fluids. With the same model, Shashi Goel (1964) discussed the flow between two infinite disks (one rotating and the other at rest) for small values of the Reynolds number. Luthra (1964) studied the problem of an infinite rotating disk when the fluid at infinity is also rotating in the same sense as the disk. He observed that there is a boundary layer attached to the disk, and the motion approaches the uniform rotation at infinity in an oscillatory fashion. Rathna (1962) investigated the flow of a Rivlin-Ericksen fluid near a rotating disk. She started with the fluid of the type given by the equation (1.3.6) and finally solved the equations for $\mu_3 = 0$. Recently, Srivastava (1966a) pointed out that the results of Rathna are not applicable to any real fluid since for any real fluid neither of the constants in the equation (1.3.6) is zero.

(iii) Oldroyd's fluids

In 1929 Jeffreys proposed that the behaviour of dilute suspensions and emulsions at small rates of shear is characterized by a linear equation of state of the form

$$(p_{ik}+p\delta_{ik})+\lambda_1\frac{d}{dt}(p_{ik}+p\delta_{ik})=2\eta_0\left(e_{ik}+\lambda_2\frac{d}{dt}e_{ik}\right), \quad (1.3.7)$$

where η_0 is the coefficient of viscosity, λ_1 the stress relaxation time and $\lambda_2 (< \lambda_1)$ the rate-of-strain retardation time. The relaxation time λ_1 has the physical significance that, if the motion is suddenly stopped, the stresses will decay as $\exp(-t/\lambda_1)$; and the retardation time λ_2 has the significance that, if all stresses are removed, rates of shear will decay as $\exp(-t/\lambda_2)$.

In 1950 Oldroyd made a general discussion on the invariant forms of the rheological equations of state for a homogeneous continuum, suitable for application to all conditions of motion and of stress. He imposed restrictions on their form by the requirement that the equations must describe properties independent of the frame of reference, and must be consistent with the requirement that the behaviour of the material element depends only on its previous rheological history and not in any way on the state of neighbouring elements, or on the motion of the material as a whole in space. Confining attention to rheological equations of state which are linear in the stresses alone and include terms of the second degree in the stresses and rates of strain taken together, Oldroyd (1958) showed that a valid generalization of the equation (1.3.7) is

$$p'_{ik} + \lambda_1 \frac{\mathcal{D}}{\mathcal{D}t} p'_{ik} + \mu_0 p'_{jj} e_{ik} - \mu_1 \left(p'_{ij} e_{jk} + p'_{jk} e_{ij} \right) + \nu_1 p'_{jl} e_{jl} \delta_{ik}$$

$$= 2\eta_0 \Big(e_{ik} + \lambda_2 \frac{\mathcal{D} e_{ik}}{\mathcal{D}t} - 2\mu_2 e_{ij} e_{jk} + \nu_2 e_{jl} e_{jl} \delta_{ik} \Big), \quad (1.3.8)$$

where $\mu_0, \mu_1, \mu_2, \nu_1$ and ν_2 are five more constants, each with the dimensions of time. The material derivative denoted by $\frac{\mathcal{D}}{\mathcal{D}t}$ is a total time derivative following a typical fluid element, which takes into account the linear and angular motion of the element, and for any tensor b_{ik} is expressed as

$$\frac{\mathcal{D} b_{ik}}{\mathcal{D}t} = \frac{\delta b_{ik}}{\delta t} + v_j v_{ik,j} + \omega_{ij} b_{jk} + \omega_{kj} b_{ij}, \quad (1.3.9)$$

where v_i is the velocity vector, $\omega_{ik} = \frac{1}{2}(v_{k,i} - v_{i,k})$ the vorticity tensor and a comma denotes covariant differentiation.

The general relation (1.3.8) gives the liquids A and B of Oldroyd (1950) when

$$\eta_0 > 0,\ \lambda_1 = -\mu_1 > \lambda_2 = -\mu_2 \geqslant 0,\ \mu_0 = \nu_1 = \nu_2 = 0\ (A), \quad (1.3.10)$$

and $$\eta_0 > 0,\ \lambda_1 = \mu_1 > \lambda_2 = \mu_2 \geqslant 0,\ \mu_0 = \nu_1 = \nu_2 = 0\ (B), \quad (1.3.11)$$

Oldroyd (1950) showed that the behaviour shown by the two classes of liquids A and B is different and that liquid B exhibits properties nearer to the known experimental properties of non-Newtonian fluids. Oldroyd et al. (1951) pointed out that a dilute solution of highly polymerized methyl methacrylate in pyridine behaves like an elastico-viscous liquid which in a simple shearing motion can be characterized approximately, at sufficiently small rates of shear, by the relation (1.3.7).

In 1959 Sharma investigated the rotation of an infinite plane lamina in Oldroyd's elastico-viscous liquids. He found that the elasticity of the liquid accelerates the centrifugal action near the lamina and retards it away from it. He and Srivastava (1963) discussed later the torsional oscillations of an infinite plate in the same class of fluids. The analogous problem in which the fluid is bounded by another stationary disk was studied recently by Frater (1964).

(iv) Coleman and Noll's second order fluids

In 1960 Coleman and Noll proposed the constitutive equation for an incompressible second order fluid as

$$p_{ik} = -p\delta_{ik} + \mu_1 A_{(1)ik} + \mu_2 A_{(2)ik} + \mu_3 A_{(1)i\alpha} A_{(1)\alpha k} , \qquad (1.3.12)$$

where $A_{(1)ik} = v_{i,k} + v_{k,i}$,

and $A_{(2)ik} = a_{i,k} + a_{k,i} + 2v_{m,i} v_{m,k}$,

and μ_1 , μ_2 and μ_3 are respectively the coefficients of viscosity, elastico-viscosity and cross-viscosity.

The constitutive equation (1.3.5) or Rivlin-Ericksen fluids reduces to (1.3.12) when squares and products of $A_{(2)ik}$ are neglected and the coefficients of the remaining terms are taken to be constants. Markovitz and Coleman (1964) proved that μ_2 is negative and μ_3 is positive. The solution of poly-iso-butylene in cetane behaves as a second-order fluid and the material constants for this solution at various concentrations were determined experimentally by Markovitz and Brown (see Truesdell 1964). For a 5.4%

solution of poly-*iso*-butylene in cetane at 30°C, it is found that $\mu_1 = 18.5$ poises, $\mu_2 = -0.29/cm$ and $\mu_3 = 1.0\ g/cm$.

Using the constitutive equation (1.3.12), Srivastava studied the flow due to the torsional oscillations of an infinite plate (1963), the flow between two infinite rotating disks (1964) and the flow between torsionally oscillating and steadily rotating disks (1966b). For the same class of fluids, S.K.Sharma and H.G.Sharma (1965,1966) investigated the flow over an enclosed rotating disk and the flow near a rotating plate. Other investigations in these fluids include those of Sharma et al.(1966) on the effects of injection on the flow between two infinite disks (one rotating and the other at rest) and those of G.C.Sarma (1967) on the forced flow against a rotating disk and the flow between two infinite slowly rotating disks.

1.4. OUTLINE OF THE THESIS

In the light of the survey made above, the following problems are investigated in the remaining chapters of the thesis.

In Chapter II, the problem of unsteady flow between two infinite rotating disks is studied for small values of the Reynolds number. In Chapter III, the influence of magnetic field and suction on the torsional oscillations of an infinite plate is discussed. Chapter IV is devoted to the study of the hydromagnetic forced flow against a porous rotating disk. In Chapter V, the analysis of forced flow of an Oldroyd's elastico-viscous liquid against a rotating disk is presented, and in Chapter VI, finally, the slow rotation of two coaxial infinite disks in an Oldroyd's elastico-viscous liquid is discussed.