Abstract

This thesis, as suggested in its title, analyses geometric flows of various types with and without higher order and higher derivative terms. The manifolds on which such flows are studied range from simple two dimensional ones to three dimensional homogeneous spaces as well as unwarped and warped product spaces (spacetimes) in diverse dimensions with Euclidean (Lorentzian) signature.

We begin with Ricci flow on unwarped products, wherein exact solutions, fixed curves and other generic features are pointed out. Subsequently, we look at flows on specific warped product manifolds of the kind that arise in the bulk-brane models in higher dimensional gravity theories. Here, we demonstrate how the flow equations can be separated and solved to give rise to conformally anti de Sitter spacetime. We also investigate the non–separable case numerically and obtain a 'phase diagram' separating singular and non-singular solutions.

Moving on to three dimensions and also to flows with higher order terms, we provide a detailed analysis of such flows on homogeneous spaces. Several special cases are solved analytically, others are dealt with, either numerically or by using analytical estimates. We also provide phase plots and study the evolution of the scalar curvature along the flow.

Next, we introduce higher order as well as higher derivative terms, following the β function equations for RG flow in the bosonic nonlinear σ model. We first investigate flows upto fourth order in α' for simple toy examples in two dimensions. Then, we move on to the warped products discussed earlier and show how conformally anti de Sitter spacetime can arise as a solution of the flow equations upto fourth order.

Finally, we introduce a new geometric flow, the Bach flow where we have higher order and higher derivative terms. We illustrate Bach flow on (2,2) unwarped products by solving the flow equations and analysing the fixed points. Turning to warped products, we choose a family of metrics for which the flow equations are reduced to a solvable dynamical system. We end by discussing geometric flows on a generalised and scaled family of Nariai type metrics.

In conclusion, we hope that our efforts will throw some light on (a) the nature of higher order and higher derivative flows and (b) the generic features of geometric flows on product manifolds.

Keywords: Ricci flow, Bach flow, RG flow, higher order higher derivative flow, homogeneous spaces, warped and unwarped manifolds and non linear σ-model.