

## Abstract

This report discusses a level set formulation for numerical simulation of multiphase fluid flow systems involving sharp interfaces. A mathematical model for immiscible, two-fluid system in a moving container is developed by assuming that the flow is incompressible and viscous. Surface tension force at the fluid interface is implemented through the CSF model of Brackbill *et al.* [25]. The interface between the two fluids is identified as the *zero level set* of a smooth scalar function and can be recovered at any point of time during the computation. The important aspect of re-initialization of the level set function while marching forward in time is emphasized.

The incompressible Navier-Stokes equations were solved on a staggered grid using an explicit predictor-corrector projection method. A fifth-order accurate WENO [106] scheme was used for advecting the level set function as well as reinitializing it. The implementation of WENO scheme was improved by staggering the level set function. The Navier-slip condition is applied at the point where the interface meets the solid wall. The Navier-Stokes part of the code was validated by computing the standard lid-driven cavity flow and Taylor-Green vortex. A systematic study was carried out to assess the level set methodology for capturing interfaces between two immiscible fluids by exposing the interface to a variety of complex velocity fields. The Young-Laplace law for a static drop has been verified to validate the implementation of surface tension force and the parasitic currents associated with surface tension model has been quantified.

As the level set function does not follow any physical conservation law, it inherently does not conserve the mass of individual fluid phases. Our numerical experiments show that for certain type of problems the conservation is a crucial issue with level set method even when higher order discretization schemes are used for the solution of level set and reinitialization equations. In the present work a new volume-reinitialization scheme has been proposed which when applied preserves the individual fluid volumes within the prescribed tolerance limit. The effectiveness of the method is demonstrated by simulating a wide range of two-fluid systems including, the determination of equilibrium shape of free surface in a rotating cylinder, zero-gravity drop oscillations, surface tension induced evolution of a starfish interface, drop deformation in an extensional flow field, buoyancy driven bubble motion, and small amplitude oscillation of liquid in partially filled containers.

It was observed that for certain problems, during the course of simulation, a nonphysical interface develops along the solid wall due to ineffectiveness of the standard reinitialization procedure. This introduces additional error in the solution. We have proposed and implemented a remedy for avoiding the development of false interfaces.

Diverse physical problems involving two-phase flow systems including Rayleigh-Taylor instability, the impact of surge fronts, surface tension induced coalescence of droplets, drop deformation in extensional flow field, dynamics of buoyancy-driven bubble motion, and sloshing of liquid in partially filled containers with horizontal and vertical excitations are studied and the relevant results are presented. Some of these problems (coalescence of droplets, drop deformation in extensional flow field, and sloshing with vertical excitation) represent the first attempt to solve numerically using level set methodology. Whereas, few other problems such as the simulation of bubble in wobbling regimes and simulation of large amplitude Faraday waves represent, to the best of our knowledge, the first attempt to numerically simulate the problem using full nonlinear Navier-Stokes equations. We have thus demonstrated the versatility of the level set method in solving flows with moving interfaces.

**Key Words:** Level Set Method, Two-Fluid Flows, Moving Interfaces, Unsteady Navier-Stokes Equation, Weighted Essentially Non-Oscillatory Scheme (WENO-5), Volume-Reinitialization Scheme, Rayleigh-Taylor Instability, Drop Deformation, Drop Coalescence, Bubble Dynamics, Sloshing of Liquid.