Abstract

Accurate modeling of many physical systems often leads to a set of fractional differential equations. This work deals with the optimal control of those systems. Fractional derivatives considered here are defined in the sense of Riemann-Liouville or Caputo. The performance index considered is a function of both the state and the control variables, and the dynamic constraint is described by a fractional differential equation. The variational approach, the Lagrange multiplier, and the fractional integration by parts are used to obtain the necessary conditions of optimality. To solve the resulting equations, fractional derivatives are approximated using Grünwald-Letnikov definitions. This leads to a set of algebraic equations that can be solved using numerical techniques.

Different cases of optimal control with fixed/free final time/state are considered. A general numerical method is developed for solving the problems which could be defined in the Riemann-Liouville or the Caputo derivative sense. Numerical examples are provided to show the effectiveness of the formulation and the numerical schemes. Results show that if the fractional derivatives are made equal to the integer derivatives, the numerical solutions agree with the analytical solutions.

This work also presents a pseudo-state-space representation of the fractional dynamic systems. The present pseudo-state-space representation is finite dimensional and it recovers integer state-space if the fractional derivatives are made equal to integer derivatives. Moreover, based on this representation an optimal control problem is formulated. A numerical technique using Grünwald-Letnikov definitions is used to obtain the state and the control variables.

Keywords: fractional derivative, optimal control, state-space, two-point boundary value problem

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