Abstract

This thesis is concerned with the spectral properties of eccentricity matrices of graphs. The eccentricity matrix of a connected graph G, denoted by $\mathcal{E}(G)$, is constructed from the distance matrix of G by keeping only the largest entries in each row and each column and leaving zeros in the remaining ones. The $\mathcal{E}$-eigenvalues of G are the eigenvalues of $\mathcal{E}(G)$, in which the largest one is the $\mathcal{E}$-spectral radius of G. The eccentricity energy (or the $\mathcal{E}$-energy) of G is the sum of the absolute values of all $\mathcal{E}$-eigenvalues of G.

In the first chapter, we give a background of the thesis and describe some of the needed concepts from graph theory and spectral graph theory. Also, we define the useful notations and collect the necessary results along with a brief overview of the literature to set the stage for the study conducted in the subsequent chapters. The motivation and objectives of the thesis as well as the organization of the thesis, are also given in this chapter.

In Chapter 2, we focus on the inertia and symmetry of eigenvalues of the eccentricity matrices of trees. For a tree T with an odd diameter, first, we prove that the rank of the eccentricity matrix $\mathcal{E}(T)$ of T is 4. Moreover, we show that $\mathcal{E}(T)$ has exactly two positive and two negative eigenvalues irrespective of the structure of the tree T. Also, we prove that any tree with an even diameter (except the star graph) has an equal number of positive and negative $\mathcal{E}$-eigenvalues, which is equal to the number of 'diametrically distinguished' vertices in the tree. Then, we show that the $\mathcal{E}$-eigenvalues of a tree are symmetric with respect to the origin if and only if the tree has an odd diameter. Furthermore, we obtain an upper bound for the least $\mathcal{E}$-eigenvalue of a tree with an odd diameter. Finally, we characterize the trees with three distinct $\mathcal{E}$-eigenvalues.

In Chapter 3, we consider minimization problems for the second largest $\mathcal{E}$-eigenvalue and $\mathcal{E}$-energy of trees and characterize the extremal trees. First, we determine the unique tree with the minimum second largest $\mathcal{E}$-eigenvalue among all trees on n vertices other than the star. Then, we characterize the trees with minimum $\mathcal{E}$-energy among all trees on n vertices.

In Chapter 4, we consider the extremal problems for the eccentricity matrices of complements of trees and characterize the extremal graphs. First, we determine the unique tree whose complement has minimum (respectively, maximum) $\mathcal{E}$-spectral radius among the trees with a fixed number of vertices. Then, we prove that the $\mathcal{E}$-eigenvalues of the complement of a tree are symmetric about the origin. As a consequence of these results, we characterize the trees whose complement has minimum (respectively, maximum) least $\mathcal{E}$-eigenvalues among the complements of trees. Finally, we discuss extremal problems for the second largest $\mathcal{E}$-eigenvalue and the $\mathcal{E}$-energy of complements of trees and characterize the extremal graphs. As an application, we obtain a Nordhaus-Gaddum type lower bounds for the second largest $\mathcal{E}$-eigenvalue and $\mathcal{E}$-energy of a tree and its complement.

In Chapter 5, we study the effect of edge deletion on the energy of eccentricity matrices of graphs. First, we provide examples of graphs for which the eccentricity energy increases, decreases, or remains the same due to an edge deletion. Then, we prove that the eccentricity energy of a complete k-partite graph $K_{n_1,\ldots,n_k}$ with $k \geq 2$ and $n_i \geq 2$, always increases due to an edge deletion.

Finally, in Chapter 6, we derive the conclusion and describe some problems for future research based on this thesis.

Keywords: Eccentricity matrix, Tree, Complement of tree, Inertia, $\mathcal{E}$-spectral radius, Second largest $\mathcal{E}$-eigenvalue, Least $\mathcal{E}$-eigenvalue, $\mathcal{E}$-energy, Edge deletion, Complete multipartite graph.