Abstract

The thesis is a study of different dimensional parameters of graphs and partially ordered sets (posets). The research in dimension theory thrives on establishing relationships among various dimensional parameters of graphs and posets and bounding dimensional parameters in terms of other well-known and useful parameters of graphs and posets. One of the motivations behind studying any dimensional parameter is if a graph or a poset is bounded in terms of a particular dimensional parameter then many NP-hard problems can either be solved in polynomial time or be well approximated for the graph or the poset.

The *dimension* of a poset $\mathcal{P}$ is the minimum number of linear orders whose intersection is $\mathcal{P}$. Given a poset $\mathcal{P}$, a *containment model* of $\mathcal{P}$ maps every element of $\mathcal{P}$ into some geometric or graph object in such a way that for every two distinct elements x, y of $\mathcal{P}$, x is related to y in $\mathcal{P}$ if and only if the object that is representing x is completely contained inside the object that is representing y. The circle order, n-gon order, interval containment order are some examples of geometric containment models whereas containment orders of subgraphs in a host graph, paths in a host tree, subtrees in a host tree are some examples of graph containment models. We study the *Containment order of Paths in a Tree (CPT) poset* which was introduced by Corneil and Golumbic [1]. We show an upper bound for the dimension of CPT posets in terms of the maximum degree and radius of the host tree. This bound is asymptotically tight up to a very small additive factor. The proof of our main theorem gives a simple algorithm to construct a realizer for the standard poset $\mathcal{P}(1, 2; n)$ i.e. the poset consisting of all the 1-element and 2-element subsets of $[n]$ under containment relation.

The *intersection dimension* of a graph G corresponding to a graph class $\mathcal{A}$ (also called $\mathcal{A}$-*dimension* of G) is the minimum positive integer k such that G is the intersection of k graphs in $\mathcal{A}$. This framework was introduced in [Kratochvíl and Tuza, Graphs and Combinatorics, 1994]. The graph parameters generated from intersection dimension corresponding to various classes of graphs are sometimes referred to as *dimensional parameters* of a graph. Representing graphs as the intersection of graphs taken from a particular graph class is a well-studied topic in graph theory and combinatorics. The *boxicity*

[1] Unpublished but cited in [Golumbic, IBM Scientific Center T. R., 1984] and [Golumbic and Scheinerman, Annals of the New York Academy of Sciences, 1989]

of a graph G is the intersection dimension of G corresponding to the class of interval graphs. We study the local boxicity of a graph, which is a variation to the classical graph dimensional parameter, boxicity. It is a comparatively new dimensional parameter in the literature which formally appeared in [Bläsius, Stumpf, and Ueckerdt, Discrete Mathematics, 2018]. Local boxicity is proven to give a space efficient representation of a graph over boxicity. The *local boxicity* of a graph G is the minimum positive integer ℓ such that G can be obtained using the intersection of k (where $k \geq \ell$) interval graphs where each vertex of G appears as a non-universal vertex in at most ℓ of these interval graphs. We show upper bounds for the local boxicity of a graph in terms of the maximum degree, order, size, product dimension, etc. of the graph. All the results for local boxicity automatically extend to local dimension (the local counterpart of poset dimension) due to the known connection between the local dimension of a poset and the local boxicity of its underlying comparability graph (the comparability graph $G_{\mathcal{P}}$ of a poset $\mathcal{P}$ is the graph whose vertex set has a bijective map to the set of elements of $\mathcal{P}$ and two vertices are adjacent in $G_{\mathcal{P}}$ if and only if they are mapped to elements that are comparable in $\mathcal{P}$).

We study another dimensional parameter of a graph namely threshold dimension. The *threshold dimension* of a graph G, introduced in [Chvátal and Hammer, Annals of Discrete Mathematics, 1977], is the intersection dimension of G corresponding to the class of threshold graphs. The class of threshold graphs forms a proper subclass of the class of interval graphs leading to the observation that the boxicity of a graph is at most its threshold dimension. Though the class of threshold graphs is thoroughly studied, not much literature is available around the threshold dimension of a graph which is one of our major motivations to study this dimensional parameter of graphs. Given a graph G on n vertices, $f_G \colon \{0,1\}^n \to \{0,1\}$ is the Boolean function such that $f_G(x) = 1$ if and only if x is the characteristic vector of a clique in G. A Boolean function f for which there exists a graph G such that $f = f_G$ is called a *graphic* Boolean function. Interestingly, the threshold dimension of a graph G is the minimum number of majority gates whose AND (or conjunction) realizes the graphic Boolean function corresponding to G. We show tight or nearly tight upper bounds for the threshold dimension of a graph in terms of its treewidth, maximum degree, degeneracy, number of vertices, size of a minimum vertex cover, etc. We also study threshold dimension of random graphs and graphs with high girth.