

# ABSTRACT

Let  $G$  be a graph and  $I = I(G)$  be its edge ideal. We give an explicit description of the symbolic powers and compute the Waldschmidt constant for the edge ideal  $I = I(G)$  when  $G$  is the clique sum of two different length odd cycles joined at single vertex. We prove that regularity of  $(n+1)$ -th ordinary and symbolic power is equal for the edge ideal of same class of graphs. For a complete graph  $G$ , we describe the generators of the symbolic powers of the edge ideal  $I$  and also compute the Waldschmidt constant and the resurgence. Moreover for complete graph, we show that the edge ideal  $I$  satisfies  $\dim I^{(s)}/I^s = 0$ , and using this fact we prove that Minh's conjecture is true for complete graph i.e. the Castelnuovo-Mumford regularity of the symbolic powers and ordinary powers of the edge ideal coincide. We construct a more general class of graph  $G_{n,r}$ , where  $G_{n,r}$  denote the graph with  $n$  vertices  $\{x_1, \dots, x_n\}$  in cyclic order and for each vertex  $x_i$  consider the set  $A_i = \{x_{i-r}, \dots, x_{i-1}, x_{i+1}, x_{i+2}, \dots, x_{i+r}\}$ , where  $x_{i-j}$  is the vertex  $x_{n+i-j}$ , whenever  $i < j$  and  $0 \leq r \leq \lfloor \frac{n}{2} \rfloor - 1$ . In  $G_{n,r}$ , every vertex  $x_i$  is adjacent to all the vertices of  $V(G_{n,r}) \setminus A_i$ . Let  $I = I(G_{n,r})$  be the edge ideal of  $G_{n,r}$ , we observe  $I$  satisfies  $\dim I^{(s)}/I^s = 0$ . We show that Minh's conjecture is true for  $I$ . We compute the Waldschmidt constant and resurgence for the whole class. We investigate the Castelnuovo-Mumford regularity of ordinary powers, integral closure of powers and symbolic powers of squarefree monomial ideals for certain cases. We provide a lower bound for regularity of all the ordinary powers, symbolic powers and integral closure of powers of edge ideals of hypergraphs in terms of induced matching number. We show that for a square-free monomial ideal  $I$ , if  $\dim I^{(s)}/I^s = 0$  for all  $s \geq 1$  then  $\text{reg } I^{(s)} \leq \text{reg } \overline{I^s} \leq \text{reg } I^s$  and if  $\dim I^{(s)}/I^s = 1$  then  $\text{reg } I^{(s)} \leq \min\{\text{reg } I^s, \text{reg } \overline{I^s}\}$ . We investigate Minh's conjecture for the second power and using local Cohomology prove that if  $\dim I^{(2)}/I^2 \leq 1$  or  $G$  is gap free then  $\text{reg } I^2 = \text{reg } I^{(2)}$ . We show that the regularity of ordinary powers and symbolic powers of edge ideals of complete  $d$ -uniform hypergraphs is equal.

**Keywords:** symbolic power, regularity, edge ideal, Waldschmidt constant, resurgence number, symbolic defect, integral closure, induced matching number, co-chordal number,