Abstract

The present work focuses on dynamic analysis of parametrically excited microcantilever beam used for mass sensing. Microcantilever beams are widely used for mass detection. The change in the dynamic properties of the beam due to any additional mass is sensed for the purpose of mass detection. In this thesis, the beam is excited for parametric resonance, rather than to resonance by direct forcing. Here, parametric excitation is achieved by displacing the base of the cantilever beam in the transverse direction with the help of a feedback mechanism. The system studied in this thesis is nonlinear. The nonlinear terms appear both in the equation of motion of the moving cantilever beam and in the feedback signal generated from the displacement sensor, which is an integral part of the system. It is found in the study that the presence of nonlinear terms can enhance the performance of the system for mass detection. For better sensing, nonlinearity can even be introduced in the feedback path.

The dynamics of the system has been studied analytically, using the multiple-time scales method and also using different techniques of numerical analysis. The study has been carried out by neglecting the nonlinear terms initially and again by including them. A novel analytical method has been proposed to determine the stability boundaries of the parametrically excited linear system. An extension of the method is deployed for determining the stability boundaries of the system under degenerate parametric excitation. In both the studies, the results obtained through the analytical method yielded a very good match with those obtained from Discrete Element Method (DEM). The nonlinear response of the cantilever beam has been estimated by the multiple-time scales method. The studies have been carried out separately for the system where the nonlinear terms are present only in the feedback path and when the terms are present both in the feedback loop and within the equation of motion of the cantilever beam. For the former case, the system response has also been obtained using DEM. For the latter, on the other hand, a differential quadrature method and Galerkin method have been used to find the response numerically.

A couple of mass sensing strategies have been proposed in the thesis. In the first one, linear parametric resonance has been utilized. For this case, observing the power spectrum of the time response, it is found that through measuring the shift of Fourier coefficients at higher harmonics, sensitivity can be improved up to 3 to 7 times. For another proposed mass detection approach, a large slope of amplitude-frequency curve has been found helpful. Here, change in amplitude of the response is calculated to determine the added mass. Through perturbation and numerical analysis, it has been confirmed that the proposed methods can provide better sensitivity in comparison to the conventional MEMS resonator-based mass sensing methods. Further, the noise characteristics of the parametrically excited system are better compared to the directly excited one. This has been confirmed by numerical analysis.

Keywords: Resonant microcantilever beam; parametric excitation; feedback; nonlinear response; mass sensing; noise.