Abstract

A wide range of nonlinear physical phenomena in the vast areas of scientific disciplines are depicted by the system of nonlinear coupled partial differential equations (PDEs). These systems describe multiple behaviors in various fields such as mathematical physics, fluid dynamics, chemistry, condensed matter, biophysics, plasma physics, optical fibers, biology and other areas of engineering. The exact solutions of such system of nonlinear PDEs play an important role in nonlinear science, especially in nonlinear physics, since they can yield very much physical information and more insight into the physical aspects of the problem and thus lead to applications like understanding the behavior of the physics associated with the problem. Lie group analysis, which is based on invariance principle, is the only method which can be applied systematically to such nonlinear system of PDEs to obtain exact solutions.

The concept of optimal classification of Lie point symmetries is very significant as the search of group invariant solutions can be minimized with the help of an optimal set. But, the study of optimal set of subalgebras for the infinite dimensional Lie algebra is less explored and thus require research attraction. We study one-dimensional optimal classification of (2+1)-dimensional Boiti-Leon-Pempinelli system of nonlinear PDEs and obtain some new exact solutions. Then we perform nonclassical analysis (generalization of classical symmetry) to the one-dimensional macroscopic production model to obtain some exact solutions. Moreover, several conservation laws are constructed by using multipliers method and also nonlinearly self-adjointness principle.

Apart from local symmetries, in the recent past the concept of nonlocal symmetries have become very popular and challenging in the area of symmetry analysis. There are two ways to compute the nonlocal symmetries, conservation laws based method and symmetry based method. We have presented a systematic algorithm to obtain exact solutions of any given PDEs by using nonlocal symmetries arising from symmetry based method. We have performed nonlocal symmetry analysis, arising from both potential systems as well as inverse potential systems, to an integrable soliton equation and also to a system of PDEs that governs one-dimensional macroscopic production model. Some new nonlocal conservation laws are established for these systems.

Nonclassical potential symmetries are symmetries obtained by performing nonclassical method to the potential system or sometimes the potential equation corresponding to the given system. These yield some new exact solutions those can not be obtained by using either classical symmetries or conditional symmetries or even potential symmetries. We have obtained several new exact solutions by performing the nonclassical potential symmetry analysis to a system of PDEs that governs the dynamics of a thin film model of a perfectly soluble anti-surfactant solution. Moreover, as an application of exact solutions, we have studied some interesting phenomena like, evolution of characteristic shock, weak discontinuity

and interactions between them.

Throughout the thesis, we have studied some of the obtained exact solutions in terms of their physical significance which includes several soliton solutions.

Keywords : Lie group of transformations; Nonclassical analysis; Nonlocal symmetry; Conservation laws; Solitons; Boiti-Leon-Pempinelli system; Integrable soliton equation; Macroscopic production model; Thin film model; Weak discontinuity; Characteristic shock; Wave interactions.