Abstract of Thesis

Nonlinear differential equations model numerous physical phenomena in applied science and engineering. Therefore they play an essential role in real-world applications. Generally, the evaluation of exact solutions to these problems poses a significant challenge. More precisely, the exact solutions are only available in a few cases. Thus, the construction of effective methods to solve them is crucial. Therefore, over the past decades, there has been considerable attention developing various iterative methods to obtain an analytical approximation to the solution of various ordinary and partial differential equations. Most of these methods have multiple shortcomings and are computationally intensive due to complicated symbolic computations. Thus, the main objective of this study is to develop computationally efficient iterative methods to solve nonlinear differential equations effectively.

The aim of this thesis is twofold. First, we introduce an effective approach, which is based on the combination of Newton's quasilinearization and Picard iteration method to solve nonlinear differential equations effectively. In this approach, the quasilinearization technique is utilized to reduce the nonlinear problem to a sequence of linear problems, and then the Picard iteration method is used to solve the linearized problems arising from quasilinearization. Secondly, we introduce iterative methods based on Green's function and some well-known fixed point iteration schemes, such as Halpern and Normal-S iterative schemes, to approximate the solution of boundary value problems.

The content of the thesis is divided into seven chapters. Chapters 1 is the introductory part of the thesis in which a general introduction, a brief literature survey, and mathematical preliminaries are discussed. In chapter 2, we develop the quasilinearized Picard iteration method, which is the combination of Newton's quasilinearization method and Picard iteration method to obtain an analytical approximation to the solution of a class of two-point nonlinear doubly singular boundary value problems arising in various physical models. Further, it is shown that this method also works quite efficiently for other nonlinear problems by successfully implementing the method on second, third, and fourth-order regular boundary value problems. The applicability of this method has also been tested for nonlinear partial differential equations by implementing the method on the well-known Klein-Gordon equations. In chapter 3, we introduce the optimal quasilinearized Picard iteration method, which is the combination of optimal quasilinearization method and Picard iteration method or optimal Picard iteration method to obtain an analytical approximation to the solution of nonlinear differential equations including the well-known Bratu's problems. In chapter 4, we introduce the parametric quasilinearized Picard iteration method to solve a nonlinear fourth-order two-point boundary value problem with nonlinear boundary condition involving third-order derivative which models an elastic beam on the elastic bearing. Further, it is shown that this method also works quite effectively for other nonlinear boundary value problems associated with nonlinear boundary condition which is demonstrated by applying the proposed to a second-order singular boundary value problem and a third-order nonlinear boundary value problem with nonlinear boundary condition. In chapter 5, we introduce the piecewise quasilinearized Picard iteration method for solving nonlinear initial as well as boundary value problems including the well-known Riccati differential equations, Bratu's equations, and Lane-Emden type equations on a larger interval. In chapter 6, we derive two analytical iterative methods based on Halpern and Normal-S iterative schemes for solving a class of two-point nonlinear singular boundary value problems and a class of nonlinear fourth-order two-point boundary value problems. Moreover, the convergence analysis of the proposed methods is also discussed in the thesis. Chapter 7 deals with the conclusion as well as the future scope of the work carried out in this thesis.

To demonstrate the efficiency, robustness, and applicability of the proposed methods, we consider various numerical examples, including real-life problems. This is crucial, as shown in multiple numerical tests where our new proposals outperform the existing methods. The numerical simulations illustrate that the proposed methods are straightforward to implement and minimize the computational work compared with the other methods. These methods are computationally efficient and overcome numerous shortcomings of the existing methods. In fact, the proposed methods are an advanced approach for dealing with different types of highly nonlinear problems, which provide a highly accurate approximate solution to the problems and require a few iterations to obtain reasonably good accuracy at a reasonable computational cost.

Keywords: Quasilinearization method, Picard iteration method, Nonlinear differential equations, Boundary value problems, Initial value problems, Nonlinear boundary conditions, Green's function, Iterative methods, Convergence analysis, Analytical approximate solution