Abstract

In this thesis, we consider projection and multi-projection methods to approximate the solutions of linear and nonlinear Fredholm integral equations of the second kind on unbounded domains (half-line/real line) using both piecewise and global polynomial basis functions. In this, first we discuss Galerkin and collocation methods and their iterated versions to approximate the solution of nonlinear Hammerstein type integral equation on the half-line for both convolution and non-convolution kernels using piecewise polynomial basis functions. We also discuss multi-Galerkin, multi-collocation methods and their iterated versions to approximate the solution of the same nonlinear Hammerstein type integral equation on the half-line and obtain the superconvergence results over Galerkin, collocation methods and their iterated versions in the space of piecewise polynomial basis functions in infinity norm. However in piecewise polynomial based methods, one has to increase the number of partition points, which leads to solve a large number of nonlinear system of equations, which is computationally very expensive. This motivate us to use global polynomials as basis functions instead of using piecewise polynomials as basis functions. We develop Galerkin, multi-Galerkin methods and their iterated versions to solve both the linear and nonlinear (Hammerstein type) Fredholm integral equations of the second kind on the half-line using Laguerre polynomials (global polynomials) as basis functions. In fact we obtain the superconvergence results in multi-Galerkin and iterated multi-Galerkin methods over Galerkin and iterated Galerkin methods in weighted L^2-norm under the same assumptions as in Galerkin method. Next we establish the Galerkin, multi-Galerkin methods and their iterated versions to solve the linear Fredholm integral equation of the second kind on the real line with sufficiently smooth kernels, using Hermite polynomials as basis functions. We obtain optimal convergence rates in iterated Galerkin method in weighted L^2-norm. We also discuss multi-Galerkin, iterated multi-Galerkin methods and we obtain the superconvergence results in both multi-Galerkin and iterated multi-Galerkin methods in weighted L^2-norm. Numerical results are presented to confirm the theoretical results.

Keywords: Hammerstein integral equation, Galerkin method, collocation method, multi-Galerkin method, multi-collocation method, Piecewise polynomials, Laguerre polynomials, Hermite polynomials, Superconvergence results.