Abstract

This thesis is concerned with the detailed investigation of the statistical properties of complex systems with column/row constraints. Using random matrix approach a complex system can be represented by a generic random matrix ensemble. Here we consider the effect of a combination of matrix constraints e.g. Hermiticity and time-reversal symmetry besides column/row sum rule, as well as the ensemble constraints (e.g. disorder), on the matrix ensembles. This topic has gained rapid attention in the recent past decade because of its appearance in widely different areas e.g. bosonic Hamiltonians such as phonons, and spin waves in Heisenberg and XY ferromagnet, antiferromagnets, and spin glasses, Goldstone modes, Euclidean random matrices, random reactance networks, Internet related Google Matrix, financial markets and pattern games etc.

The thesis consists of three main parts. In the first part of my thesis, we analyze the role of a specific global constraint on complex systems, appearing as column/sum rules on their matrix representations and in combination with other local and global constraints. Our study reveals that the presence of additional constraints besides real-symmetric nature introduce new correlations among the matrix elements which, as a consequence, influence their distribution and manifest in their eigenvalues and eigenfunctions too. As physical properties, in principle, can be explained in terms of eigenvalues and eigenfunctions., therefore, we focus on the effects of constraints on the joint eigenvalue-eigenfunction distribution. The latter turns out to be analogous to that of a special type of critical Brownian ensemble without such constraint, intermediate between Poisson and Gaussian orthogonal ensemble. Another important finding of our work is the lack of single basis state localization for a typical eigenvector which is in contrast with an unconstrained real-symmetric matrix.

In the second part of the thesis, we pursue a detailed numerical investigation of the random matrix ensembles of real-symmetric matrices with column/row constraints for many system conditions e.g. disorder type, matrix-size and basis-connectivity. The results show a rich behavior hidden beneath the spectral statistics and also confirms our analytical predictions, as presented in first part of the thesis, about the analogy of their spectral fluctuations with those of a critical Brownian ensemble which appears between Poisson and Gaussian orthogonal ensemble.

A Brownian ensemble is a non-equilibrium state of transition from one universality class of random matrix ensembles to another one. The parameter governing the transition is in general size-dependent, resulting in a rapid approach of the statistics, in infinite size limit, to one of the two universality classes. In the last part of the thesis, our detailed analysis however reveals the appearance of a new scale-invariant spectral statistics, non-stationary along the spectrum, associated with multifractal eigenstates and different from the two end-points if the transition parameter becomes size-independent. The number of such critical points during transition is governed by a competition between the average perturbation strength and the local spectral density. The results obtained in my thesis have applications to wide-ranging complex systems e.g. those modeled by multi-parametric Gaussian ensembles or column constrained ensembles.
Keywords: Random matrix, fluctuations, constraints, universality, complexity parameter, Brownian ensemble, criticality.