Abstract

The present thesis deals with wave interaction problems associated with quasilinear one-dimensional hyperbolic system of conservation laws, which occurs in many physical phenomena having practical importance in real life. We investigate solution of the Riemann problem and use this solution to discuss the wave interactions for various practical problems including blood flow equations, macroscopic production model, anti-surfactant fluid and drift-flux equations of two-phase flows.

First, we consider one-dimensional blood flow equations and study the elementary waves and their properties to establish existence and uniqueness of the solution to the Riemann problem. We prove that the solution of the Riemann problem may consists of a vacuum state if the initial data does not satisfy certain bound. Moreover, we discuss some numerical examples with different initial data along with all possible interactions of elementary waves in the phase plane. Next, we consider the macroscopic production model and discuss the interaction of simple waves by considering the solution of the Cauchy problem. We use the hodograph transformation and differential constraints technique to obtain the exact solution of governing equations. Further, we discuss the stability of the solution to the Riemann problem by considering the interaction of elementary waves in the characteristic plane for the system of equations governing the dynamics of a thin film of anti-surfactant liquid. We prove that the solution of the Riemann problem is stable under the small perturbation of the Riemann initial data.

Next, we consider the widely used drift-flux model of two-phase flows. Here, we discuss the interaction of elementary waves with a weak discontinuity in the characteristic plane by considering the isothermal equation of state. We study the behavior of the amplitudes of reflected and transmitted waves depending upon the choice of initial data with numerical examples. Finally, we study the interaction of two weak shocks of the Riemann problem for the drift-flux equations with the isentropic equation of state. Here, we establish the condition on initial data such that the solution consists of shocks and contact discontinuity only and discuss the interaction between weak shocks of the same family.

Keywords: Riemann problem, Wave interactions, Stability, Simple wave, Shock wave, Rarefaction wave, Contact discontinuity, Two phase flows, Drift-flux model, Macroscopic production model, Anti-surfactant solution, Blood flow equations, Weak discontinuity.