Abstract

Most of the studies on migration of droplets and bubbles in a viscous fluid are restricted to steady axisymmetric flows and assumed linear dependency of interfacial tension on either temperature or surfactant concentration. The main objective of the present thesis is to investigate problems of migration of droplets and bubbles subject to the variation of interfacial tension due to thermal and surfactant concentration gradients. The mathematical models are based on conservation principles, which are Navier-Stokes equations, conservation of energy, convection-diffusion, and conservation of mass equations. The Stokes or transient Stokes equations are used to formulate the hydrodynamics of the flow inside and outside a droplet. We solve the arbitrary Stokes or transient Stokes equations by using a solenoidal decomposition method in which the velocity vector field can be represented in terms of two scalar functions. Hence, the hydrodynamic problem simplifies to handling these two scalar functions. A detailed understanding of effect of temperature and surfactant concentrations on the droplet migration is discussed. Chapter 1 of the thesis is introductory.

We basically solve the thermal problem independently by neglecting convective terms and use these temperature fields while solving the corresponding hydrodynamic problem. We couple the thermal and hydrodynamic problems via the tangential stress balance boundary condition. In chapter 2, we solve the heat conduction problem in steady and transient cases and discuss the responses due to various ambient thermal fields. The role of temperature field on the hydrodynamics of the droplet is analyzed in chapter 3 as a first step. The drag force and torque are computed in terms of Faxen's laws and the corresponding migration velocity is expressed in a closed form. This would enable one to design parameter combinations to control the droplet migration for a possible use in various applications. In chapter 4, some applications and examples are discussed to understand the importance of the obtained generalized results.

In chapter 5, the migration of a surfactant coated droplet in a transient Stokes flow is analyzed under the influence of interfacial slip. In particular the regime of low surface Péclet number is analyzed using a perturbation method in powers of the surface Péclet number. The surface equation of the deformed sphere has been determined by an iterative method up to the first order approximation. Analytical expressions for the migration velocity of the droplet are likewise given. Based on this analysis we can completely characterize various flow situations like when the ambient flow is uniform flow or Couette flow or Poiseuille flow. Moreover, our computed results show that a surfactant-induced cross-stream migration of a droplet occurs towards the center-line in both, Couette and Poiseuille flow cases. Finally, the theoretical findings are validated with available experimental data.

Subsequently, the migration of a surfactant coated droplet in a non-isothermal field is studied in chapter 6. In this chapter, we consider a more generalized nonlinear state of equation for the interfacial tension relating temperature and surfactant

concentration. However, we consider the steady Stokes flow approximation. We analyze the problem analytically in two limiting cases namely low and high surface Péclet number avoiding intermediate range. We derive closed form expressions for the drift and the migration velocity where the capillary stresses can be non-axi symmetric and along axial or transverse direction. Since the results are for any arbitrary ambient flow, we have provided the corresponding analysis when the ambient hydrodynamic flow is due to Poiseuille flow. The corresponding results when the thermal gradients are along axial or transverse to the flow direction are discussed. A significant impact of the nonlinear variation is seen on the droplet axial and cross migration. We observe that the droplet can move towards or away from the center line depending on the critical thermal Marangoni number that can be estimated as a function of other parameters involved.

Finally in chapter 7, we analyze the problem of collection of a polydisperse spherical droplets. In this chapter, we assume that the collection of spherical droplets behave as a porous medium and estimate the hydrodynamic interactions analytically via the so called cell model that is defined around a specific representative particle. We estimate the overall bed permeability and show possible comparisons with Carman-Kozeny relation.

Keywords : Droplets and bubbles; Surfactant effects; Thermocapillary; Solutocapillary; Marangoni phenomena; Droplet Migration; Stokes flow; Transient Stokes flow; Capillary flows; Interfacial flows; cell model.