Chapter 1

Introduction

1.1. Background and Motivation

In an ideal vibrating system, excitation (energy source) is not influenced by the response of the system. On the contrary, when the excitation is influenced by the response of the system, the excitation is said to be a non-ideal source and overall system is a non-ideal vibrating system. All dynamical systems are inherently non-ideal. In recent times, study of the non-ideal dynamical systems has been envisaged as a major challenge in engineering research. In non-ideal vibrating systems, power supply is limited. In terms of mathematical modeling, an additional equation is required to describe how the energy source interacts with the rest of the dynamical system. More precisely, to describe a non-ideal dynamical system model one needs to have an additional degree of freedom compared to its ideal counterpart. To the knowledge of the author, compared to the case of ideal vibrating systems, the available research work for non-ideal vibrating systems is scanty.

Arnold Sommerfeld [Sommerfeld, 1902] discovered a phenomenon, better known as the Sommerfeld effect, which is characterized by the jumps in the system response at critical values of power input to the source. He conducted an experiment consisting of a cantilever beam connected with a non-ideal energy source at its free end and detected the energy interaction between the source and its elastic foundation. It was found that during the coast up operation, as the driving frequency gets closer to the natural frequency and if the drive power increases further, the motor (non-ideal energy source) speed remains same until it suddenly jumps to a much higher value (simultaneously its amplitude jumps to a much lower value) upon exceeding a critical input power. This phenomenon is commonly known as the jump phenomenon. The same phenomenon is observed during coast down operation, when the input power is gradually reduced, *i.e.,* near the region of resonance the speed suddenly jumps to a marginally lower value and the amplitude of vibration jumps to a higher value. However, the transition points in these two operations are not necessarily identical. This kind of jump appears in the frequency response curve as a discontinuity that indicates a region where steady-state condition does not exist, *i.e.*, there are some forced response frequencies that can never be reached in steady state. Sommerfeld's observation was that the structural response of the system to which a non-ideal electrical motor is connected may act like energy sink under certain conditions so that a part of the energy supplied by the source is spent to vibrate the structure rather than increasing the drive speed.

Study of the Sommerfeld effect in rotor dynamics is of great importance in recent times. A rotor driven by an ideal energy source, *i.e.*, a source capable of delivering unlimited amount of power, becomes unstable beyond a certain threshold spin speed due to non-conservative circulatory force which is similar to follower force. The circulatory force arises out of rotating internal damping. If the drive is non-ideal then the rotor spin speed cannot exceed the stability threshold. This phenomenon leads to a type of Sommerfeld effect when the system is caught at the stability threshold.

The internal damping (also known as material damping) in rotating shafts appears due to change in flexure of the rotor shaft and also from frictions induced in spline joints used to connect long rotor shafts. This damping has to be viewed from a frame attached to the shaft. When the rotor-shaft exhibits synchronous whirl, there is no change in its flexure as viewed from a rotating frame and thus there is no effect of internal/material damping. However, if the whirl is asynchronous then material damping induces a damping force much like the external damping and in addition, a non-potential stiffness like force field called circulatory force also shows up. This circulatory force draws power from the motor and is responsible for instability of the system at speeds beyond the critical speed(s) of the system.

Usually, internal damping depends on temperature and strain rate. It also depends on the damage caused by past loading (history of stress cycle). Earlier studies

[Audenino & Calderale, 1996] have shown that internal damping is inherently a non-linear phenomenon even in some isotropic materials. The power drawn by the circulatory force(s) in a supercritical rotor is spent to vibrate the system rather than to increase the drive speed. This interferes with the safe operation of rotor dynamic systems.

So far, the influence of rotating internal damping on the Sommerfeld has not been studied by the researchers. The threshold of power input where jump from a lower speed (just above the critical speed) to a higher speed occurs may change depending on the extent of the internal damping present in the system. Moreover, if the afore-mentioned higher speed is above the stability threshold speed (where another Sommerfeld effect is observed) then a strong interaction between the two Sommerfeld effects, one due to synchronous forced response (induced by power drawn from the drive by rotating eccentric mass) and the other due to asynchronous free response (induced by power drawn from the drive by circulatory forces), is expected. The prime objective of the thesis is to understand these energetic interactions and their consequences on the system dynamics through analytical methods based on multi-energy domain bond graph models of these systems.

1.2. Contribution of the Thesis

The key contributions of this work are summarized as follows:

- It is shown that the stability threshold may restrict the jump phenomena owing to the Sommerfeld effect for larger values of internal damping. Besides, it is also shown that the Sommerfeld effect would cease to exist under certain conditions. A stability condition for various branches of the steady-state solutions is derived.
- The stability domain of an internally damped flexible spinning shaft, which is driven by a non-ideal source, is studied. It turns out that the higher transverse modes may become unstable before the lower ones under certain conditions. Moreover, the steady state amplitude of the transverse vibrations in the case the shaft spinning speed is stuck at the stability threshold is determined analytically.

- The influences of internal and external damping along with gyroscopic forces and certain other parameters on the spin rate of an eccentric spinning shaft driven by a DC motor are investigated. It is shown that the internal damping affects the system dynamics in an unexpected manner. Appearance of multiple jump phenomena in an infinite dimensional non-ideal system is also exhibited. Steady-state spinning frequency and whirl orbit amplitude for this system are analytically derived as functions of the drive and the shaft system parameters and later, they are validated through simulations.
- Bond graph models of discrete and distributed parameter rotor dynamic systems have been developed. These models are used to express the source loading terms in the equations of motion and are also used for numerical evaluation. Moreover, the models serve as templates for future research where other kinds of drives, bearings, rotor disks, etc. can be incorporated.

1.3. Organization of the Thesis

This thesis is organised in five other chapters excluding this one. The contributions made in those are briefly described in the following.

A comprehensive literature survey pertaining to non-ideal systems, the Sommerfeld effect, internal damping, and bond graph modeling of rotor dynamic systems are presented in Chapter 2. In Chapter 3, Sommerfeld effect in discrete rotor dynamic systems is presented. Moreover, the bond graph models of the discrete rotor dynamic systems with and without consideration of gyroscopic effect are presented therein. The interference of the Sommerfeld effect with the stability threshold, disappearance of the Sommerfeld effect, etc. are the fundamental results presented in Chapter 3 which are the concepts that are carried on to the later chapters.

Chapter 4 details the Sommerfeld effect at stability threshold of a flexible spinning shaft driven by a DC motor. The mode that becomes unstable at the lowest shaft speed is determined in this chapter.

The Sommerfeld effect during passage through resonance of the same system considered in Chapter 4 is studied in Chapter 5.

Finally, the conclusions drawn from the thesis and the scope of future work are discussed in Chapter 6, which is followed by the list of references and appendices.