Chapter 1

Introduction

Optimal control problem is introduced first. Then optimal control problem of varieties of systems and its solution via different classes of orthogonal functions (OFs) are discussed in the literature review section. The objectives and contributions of the thesis are stated. The organization of the thesis is given in the last section.

1.1 The Optimal Control Problem

A particular type of system design problem is the problem of “controlling” a system. The translation of control system design objectives into the mathematical language gives rise to the control problem. The essential elements of the control problem are

- A mathematical model (system) to be “controlled”.
- A desired output of the system.
- A set of admissible inputs or “controls”.
- A performance or cost functional which measures the effectiveness of a given “control action”.

The mathematical model, which represents the physical system, consists of a set of relations which describe the response or output of the system for various inputs. The objective of the system is often translated into a requirement on the output. Since “control” signals in physical systems are usually obtained from equipment which can provide only a limited amount of force or energy, constraints are imposed upon the inputs to the system. These constraints lead to a set of admissible inputs.

Frequently the desired objectives can be attained by many admissible inputs, so the engineer seeks a measure of performance or cost of control which will allow him to choose the "best" input. The choice of a mathematical performance functional is a subjective matter. Moreover the cost functional will depend upon the desired behavior of the system. Most of the time, the cost functional chosen will depend upon the input and the pertinent system variables. When a cost functional has been decided upon, the engineer formulates his control problem as follows :

Determine the admissible inputs which generate the desired output and which optimize the chosen performance measure.

At this point, optimal control theory enters the picture to aid the engineer in finding a solution to his control problem. Such a solution when it exists is called an optimal control. Optimal control deals with the problem of finding a control law for a given system such that a certain optimality criterion is achieved. An optimal control is a set of differential equations describing the paths of the control variables that minimize the cost functional. The theory of optimal control is concerned with operating a dynamic system at minimum cost. The case where the system dynamics are described by a set of linear differential equations and the cost is described by a quadratic functional is called the linear quadratic problem. One of the main results in the theory is that the solution is provided by the linear-quadratic-regulator (LQR).

In this thesis different classes of systems are considered with quadratic performance criteria and an attempt is made to find the optimal control law for each class of systems using OFs that can optimize the given performance criteria.

1.2 Historical Perspective

The Legendre polynomials (LPs), originated from determining the force of attraction exerted by solids of revolution [5], and their orthogonal properties were established by Adrian Marie Legendre during 1784 - 90. There is another family of OFs known as piece-wise constant basis functions whose functional values are constant within any subinterval of time period. There is a class of complete OFs known as block-pulse functions (BPFs) which is more popular and elegant in the areas of parameter identification, analysis and control.

Control of linear systems by minimizing a quadratic performance index gives rise to a time-varying gain for the linear state feedback, and this gain is obtained by solving a matrix Riccati differential equation [3]. Probably, Chen and Hsiao, 1975 were the first, who applied a class of piecewise constant OFs i.e. Walsh functions (WFs), obtained a numerical solution of the matrix Riccati equation [6] and found the time-varying gain. Then many researchers started investigating the problems of identification, analysis and control using different classes of OFs. The operational matrix for integration of BPFs was derived [8]. Moreover, it was shown that BPFs are more fundamental than WFs and the structure of the integration operational matrix of BPFs is simpler than that of WFs. In [9] it was shown that optimal control problem could be solved using BPFs with minimal computational effort.

In the last three and half decades, OF approach was successfully applied to study varieties of problems in systems and control [21; 54; 56; 58]. The key feature of OFs is that it converts differential or integral equations into algebraic equations in the sense of least squares. So this approach became quite popular computationally as the dynamical equations of a system can be converted into a set of algebraic equations whose solution leads to the solution of the problem.

Before going into the details of optimal control problem, we first take a look at the problem of estimation of state variables as the state estimation plays an important role in the context of state feedback control. State feedback control system design requires the knowledge of the state vector of the plant. Sometimes, no state variables or only a few state variables are available for measurement. In such cases an observer, either full order or reduced order depending on the situation, is incorporated to estimate the unknown plant state variables if the plant is observable. In general it is estimated using Luenberger observer [1]. But the Luenberger observer produces erroneous estimates, in noisy environment unless the measurement noise is filtered out. Interestingly OF approach has inherent filtering property [56] as it involves integration process which has the smoothing effect. As it appears from the literature, two attempts have been made on state estimation problem by using two different classes of OFs, i.e. BPFs [22] and shifted Chebyshev polynomials of first kind (SCP1)[39] so far. It is observed that the BPF approach [22] is purely recursive and uses multiple integration. The number of integrations increases as the order of the system increases, i.e for an n^{th} order system the state equation has to be integrated n times, which is computationally not attractive.

Next coming to the SCP1 approach [39], integration operational matrix of SCP1 is less sparse than that of shifted Legendre polynomials (SLPs). So if we use SLPs to develop algorithms, it will be obviously more elegant computationally. Moreover, state estimation cannot be done via SCP1 in noisy environment.

The problem of optimal control incorporating observers has been successfully studied via different classes of OFs, namely BPFs [19], SLPs [31; 44], shifted Jacobi polynomials (SJPs) [38], general orthogonal polynomials (GOPs) [40], sine-cosine functions (SCFs) [41; 44], SCP1 [24; 44], shifted Chebyshev polynomials of second kind (SCP2) [44] and single-term Walsh series [49]. The approach followed in [24; 31; 38; 40; 41; 44] is nonrecursive while it is recursive in [19; 49], making the approach in [24; 31; 38; 40; 41; 44] computationally not attractive.

The linear-quadratic-Gaussian (LQG) control problem [4] concerns linear systems disturbed by additive white Gaussian noise, incomplete state information and quadratic costs. The LQG controller is simply the combination of a linear-quadratic-estimator (LQE), i.e. Kalman filter, and an LQR. The separation principle guarantees that these can be designed and computed independently. In [34] the solution of the LQG control design problem has been obtained by employing GOPs. By using the GOPs the nonlinear Riccati differential equations have been reduced to nonlinear algebraic equations. The set of nonlinear algebraic equations has been solved to get the solutions. The above approach is neither simple nor elegant computationally as nonlinear equations are involved.

Singular systems have been of considerable importance as they are often encountered in many areas. Singular systems arise naturally in describing large-scale systems [50]; examples occur in power and interconnected systems. In general, an interconnection of state variable subsystems is conveniently described as a singular system. The singular system is called generalized state-space system, implicit system, semi-state system, or descriptor system. Optimal control of singular systems has been discussed in [13] and [18]. In [36] the necessary conditions for the existence of optimal control have been derived. It has been shown that the optimal control design problem reduces to a two-point boundary-value (TPBV) problem for the determination of the optimal state trajectory. Single-term Walsh series method [53] has been applied to study the optimal control problem of singular systems. In [59] SLPs were used to solve the same problem. However, this approach is nonrecursive in nature. Haar wavelet approach [63] has been presented to study the optimal control problem of linear singularly perturbed systems. In the recent times, SCFs [65], SCP1 [68] and Legendre wavelets [71] have been applied for solving optimal control problem of singular systems. These approaches are again nonrecursive.

Time-delay systems are those systems in which time delays exist between the application of input or control to the system and its resulting effect on it. They arise either as a result of inherent delays in the components of the system or as a deliberate introduction of time delays into the system for control purpose. Examples of such systems are electronic systems, mechanical systems, biological systems, environmental systems, metallurgical systems, chemical systems, etc. Few practical examples [43] are, controlling the speed of a steam engine running an electric power generator under varying load conditions, control of room temperature, cold rolling mill, spaceship control, hydraulic system, etc.

As it appears from the literature, extensive work was done on the problem of optimal control of linear continuous-time dynamical systems containing time delays. Palanisamy and Rao [20] appear to be the first to study the optimal control problem via WFs. They considered time-invariant systems with one delay in state and one delay in control. In [23] time-varying systems containing one delay in state and one delay in control were considered, and optimal control problem of such systems was studied via BPFs. Solutions obtained in [20; 23] are piecewise constant. In order to obtain smooth solution, SCP1 [27] were used to study time-invariant systems with a delay in state only. In [32] time-varying systems with multiple delays in state and control have been studied via SLPs. The problem considered in [20] was investigated again by applying SLPs [35]. In [45] the problem investigated in [23] was again solved by approximating the time-delay terms via Pade approximation and using GOPs. Similarly, in [55] the problem considered in [27] was studied again via SLPs. Linear Legendre multiwavelets [70] were used to solve the time-varying systems having a delay in state only.

In the recent years, people came up with a new idea of defining hybrid functions (with BPFs and any class of polynomial functions) and utilizing the same for studying problems in Systems and Control. The so called hybrid functions approach was first introduced in [62] to study the optimal control problem of time-varying systems with a time-delay in state only. Subsequently, this approach was extended to time-invariant systems [66] studied in [27], delay systems containing reverse time terms [67], and time-varying systems [72] considered in [62]. In [73] general Legendre wavelets were used to solve the optimal control problem of time-varying singular systems having a delay in state only.

Looking at the historical developments on solving optimal control problem of nonlinear systems via OF approach, we find that not much work has been reported. Lee and Chang [37] appear to be the first to study optimal control problem of nonlinear systems using GOPs. For this, they introduced a nonlinear operational matrix of GOPs. Though their work is fundamental and significant, it is not attractive computationally as it involves Kronecker product [11] operation. Chebyshev Polynomials of first kind (CP1s) were used [47] for solving nonlinear optimal control problems. In [51] a general framework for nonlinear optimal control problems was developed by employing BPFs.

1.3 Objectives of the Thesis

In view of the above mentioned limitations of the existing works, the primary objective of this research work is to develop computationally elegant algorithms for

1. state estimation of linear time-invariant systems, and
2. solving optimal control problem of various kinds of dynamic systems with quadratic performance index

via OFs. The other objective is to develop recursive algorithms wherever possible. We consider two classes of OFs, namely BPFs and SLPs in the study of state estimation and optimal control problems.

The systems under consideration are

- linear/nonlinear,
- time-invariant/time-varying,
- time-delay free/time-delay, and
- singular/regular

systems.

1.4 Contributions of the Thesis

The major contributions of this research work can be summarized as follows :

1. In Chapter 2 two new operational matrices of reverse time are proposed; one for BPFs and other for SLPs. Also a new algorithm is proposed to calculate nonlinear operational matrix recursively by using LPs and BPFs.

2. In Chapter 3 two new recursive algorithms are presented for estimating state variables of observable linear time-invariant continuous-time dynamical systems from the input-output information using BPFs and SLPs. Also the algorithm in [22] is modified to deal with nonzero initial state vector.

3. In Chapter 4 based on using BPFs and SLPs, two recursive algorithms are presented for analysis of linear optimal control systems incorporating observers. Computational superiority of these algorithms over the existing algorithms has been discussed.

4. In Chapter 5 a unified approach and two new recursive algorithms via BPFs and SLPs are presented to solve the LQG control problem. For scalar systems a recursive algorithm via BPFs is proposed.

5. In Chapter 6 two new recursive algorithms and a unified approach are presented for computing optimal control law of linear time-invariant singular systems with quadratic performance index by applying BPFs and SLPs. The advantage of the unified approach is that it can be applicable to any singular system.

6. In Chapter 7 linear time-varying multi-delay dynamic systems are considered and presented a unified approach for computing the optimal control law and the state vector. Then this algorithm is modified to deal with time-invariant systems, delay free systems and singular systems with delays. Next linear time-varying delay systems with reverse time terms are considered and a unified algorithm for computing optimal control law for such systems is developed by using the proposed reverse time operational matrix in Chapter 2.

7. In Chapter 8 using the proposed nonlinear operational matrix in Chapter 2 and other relevant properties of LPs and BPFs, a computationally elegant method for finding the optimal control law for nonlinear systems is presented.

1.5 Organization of the Thesis

The thesis consists of nine chapters in all. They are

Chapter 1 : Introduction.

Chapter 2 : Orthogonal Functions and Their Properties; in this chapter brief discussions on OFs and their classifications are given. BPFs and their properties are given in Section 2.3. LPs are presented in Section 2.4. SLPs and their properties are discussed in Section 2.5. A nonlinear operational matrix using LPs and BPFs is introduced in Section 2.6.

Chapter 3 : State Estimation; estimation of unknown state variables is done using full order observer, both in noisy and noise free environments. The algorithm in [22] is modified to deal with nonzero initial state vector.

Chapter 4 : Analysis of Linear Optimal Control Systems Incorporating Observers; using reduced order observer the unknown state variables are estimated and the optimal control law is obtained.

Chapter 5 : The Linear-Quadratic-Gaussian Control Design; the LQG control design problem is discussed in Section 5.2. Solutions are given in Sections 5.3 and 5.4 which contain a unified approach and two recursive algorithms.

Chapter 6 : Optimal Control of Singular Systems; in Section 6.2 singular system problem is solved as an initial value problem under some conditions and two recursive algorithms are given. A more generalized solution to the problem is given in Section 6.3.

Chapter 7 : Optimal Control of Time-Delay Systems; approach for computing optimal control law of linear time-varying multi-delay dynamic systems with quadratic performance index is discussed in Section 7.2 . Then application of the approach to

- Time-invariant systems
- Delay free systems
- Singular systems with delays

is given in Subsections 7.2.3, 7.2.4 and 7.2.5. In Section 7.3 optimal control of delay systems with reverse time terms is discussed and algorithms are presented.

Chapter 8 : Optimal Control of Nonlinear Systems; a new method for computing optimal control law for nonlinear systems is given by employing LPs and BPFs.

Chapter 9 : Conclusions and Future Scope of Study; the last chapter concludes the thesis with the scope for future research.