## Abstract

This thesis proposes new and recursive computational algorithms for state estimation of linear time-invariant systems, and optimal control of various kinds of systems with quadratic performance index via orthogonal functions (OFs). Recursive algorithms are provided wherever possible. The key feature of OFs is that it converts differential or integral equations into algebraic equations in the sense of least squares. Two different classes of OFs, namely block-pulse functions (BPFs) and shifted Legendre polynomials (SLPs) are considered in the study of state estimation and optimal control problems. The systems considered are linear/nonlinear, time-invariant/time-varying, time-delay free/time-delay, and singular/regular systems.

Optimal control problem and its solution for varieties of systems via different classes of OFs are discussed. A brief discussions on OFs, their classification and important properties of BPFs and SLPs are given. New operational matrices are proposed to solve delay systems with reverse time terms, and nonlinear systems.

New recursive algorithms are presented for estimating unknown state variables of observable linear time-invariant systems, by employing full order observers via BPFs and SLPs. State estimation is also done using reduced order observers and recursive algorithms are proposed for calculating optimal control law incorporating observers.

Linear-quadratic-Gaussian (LQG) control design problem is taken up and solved it in two different ways via BPFs and SLPs. Here a unified approach and two new recursive algorithms are proposed to determine the filter gain and the regulator gain by converting the nonlinear Riccati differential equations into algebraic equations. Next linear quadratic optimal control problem of time-invariant singular systems has been considered. Again this problem is solved in two different ways, and two new recursive algorithms and a unified approach via BPFs and SLPs are proposed. The advantage and disadvantage of each method is stated.

Linear time-varying multi-delay dynamic systems are considered and a unified approach for computing the optimal control law and the state vector is presented. Later this algorithm is modified to deal with time-invariant systems, delay free systems and singular systems with delays. Then linear time-varying delay systems with reverse time terms are considered and a unified algorithm for computing optimal control law for such systems is developed. Finally optimal control of nonlinear systems is considered and a computationally attractive approach is developed to find the control law and state vector. The validity and superiority of the proposed approaches/algorithms over the existing methods are shown by solving a few examples in each case.

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**Keywords:** State estimation; Observer; Optimal control; LQG control; Quadratic performance index; Reverse time; Linear systems; Nonlinear systems; Time-invariant systems; Time-varying systems; Time-delay systems; Singular systems; Orthogonal functions; Block-pulse functions; Shifted Legendre polynomials.