

Synopsis of the thesis

**SOME STUDIES ON HYDRODYNAMICS OF FLOW
THROUGH FIXED, PARTICULATE FLUIDIZED AND
CIRCULATING FLUIDIZED BEDS**

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In Partial Fulfillment of the Requirements for the Degree of
DOCTOR OF PHILOSOPHY

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INTRODUCTION

The present thesis comprises two parts. The first part deals with studies on hydrodynamics of flow through fixed and particulate fluidized beds (liquid-solid); and the second deals with studies on hydrodynamics of flow through circulating fluidized beds (gas-solid).

PART-I: Studies on hydrodynamics of flow through fixed and fluidized bed: liquid-solid systems

A variety of mineral beneficiation, chemical and metallurgical industries deal with vertical flow / transport of complex liquid-solid systems. A number of unit operations deal with liquid-solid systems like sedimentation and hindered settling, flow through fixed and fluidized beds, co-current upward / downward flows and countercurrent flows. Generally, these are particulate flows with particles of small size of either uniform or narrow size distribution. Energy dissipation in such beds is characterized by the drag that occurs due to relative motion of fluid and particles. Drag dissipation is characterized by pressure drop and drag equations useful in estimating power requirements for movement of such fluid particle systems.

There exist a number of models describing the dissipation of mechanical energy by a fluid flowing through either a fixed or a fluidized bed. While some of them are widely accepted and have a theoretical background others stand as reliable but empirical ones upon which realistic designs can be made. But there are not many attempts at generalizing fixed bed percolation as well as fluidized bed expansion. Significant among them are the works of Barnea and Mizrahi (1973), Barnea and Mednick (1978), Ishii and Zuber (1979), Molerus (1980), Foscolo et al. (1983), Gibilaro et al. (1985), Agarwal and O'Neill (1988), Grabavcic (1992), Di Felice (1995) etc. who all, through adopting different approaches, could achieve partial success in arriving at either a coherent representation or a generalization of these two flow cases. The approaches were mainly on conduit flow, cell models etc. But their predictive expressions, equations or models are not valid for full range of porosities and particle Reynolds numbers. It has been

observed that there are limitations for the use of these equations for design purpose. Therefore, a necessity has been felt to develop generalized correlations for pressure drop, drag coefficient and drag force that are applicable to both fixed and fluidized beds. But, such correlations should be verified using experimental data and also using extensive data reported in literature.

In this work, models have been developed for flow of liquid-solid systems in fixed and fluidized beds, which have been compared with experimental data and the data collected from literature. The pressure drop and drag force predicted by various correlations reported literature were compared with those predicted by the models developed in this work.

PART-II: Studies on hydrodynamics of flow through circulating fluidized bed: gas-solid systems

Fast fluidized bed (FFB), described by Yerushalmi et al., (1985) as a dense entrained suspension, is characterized by an aggregative state in which much of the solid is, at any given moment, segregated in relatively large and densely packed groups of particles. The solid is highly turbulent and displays extensive back mixing. A bed of this nature is also called “Circulating Fluidized Bed (CFB)” because a high velocity gas is used to circulate an appropriate quantity of solid particles during the operation. Its advantage lies in the fact that, by changing the operating conditions, it is possible to maintain almost all hydrodynamic regimes of fluidized bed systems, from bubbling to pneumatic conveying with the FFB lying in between.

A circulating fluidized bed needs the creation of some special hydrodynamic conditions, namely a certain combination of gas velocity, re-circulation rate of solids, solid characteristics, volume of solids and geometry of the system, which can give rise to a state where in the solid particles are subjected to an upward velocity greater than the terminal or free fall velocity of most of the individual particles, yet these particle are not entrained immediately as observed in particle pneumatic transport systems. On the

contrary, the solids are found to move up and down in groups, known as clusters and strands. These long slender solid agglomerates are seen to move vertically, side ways as well as downwards. They are continuously formed, dissolved and reformed again.

The circulating fluidized beds can be used in the industry as chemical reactors, coal combustors for power generation and for heat transfer purposes. Since the fine particles used offer enormous surface area for gas solid contact, the rate of reaction enhances as a result of decrease in the internal resistance. Other advantages of CFB include the temperature uniformity throughout the bed, high heat and mass transfer because of the re-circulation of the solids.

In spite of the availability of a reasonable amount of literature, particularly the proceedings some of the international conferences on hydrodynamic aspects of CFB, it appears that the information on static pressure distribution and voidage distribution in the riser of a shallow CFB for low solid circulation rate is meagre.

An attempt has been made in the present thesis to obtain experimental data on axial voidage profile and static pressure distribution in the riser of a shallow, CFB with very low solid circulation rates. New approaches have also been made for the prediction of the total pressure drop in the CFB riser as the pressure drop equations reported in literature gave large deviations from the experimented data as these equations were developed for large CFB. No literature information on hydrodynamics in shallow CFB is available for the solids – phosphogypsum, rock phosphate, non-coking coal and chromite, which have been used in experiments conducted for the present work.

The thesis has been presented in the following chapters:

Chapter-I:

It describes the review of existing literature on (i) pressure drop and drag in fixed and fluidized beds separately and also the attempts for generalization of equations applicable to both the beds; and (ii) voidage profile, pressure gradient and core annulus flow in the riser of a CFB. The scope of the present work has also been highlighted.

Chapter-II:

It presents the details of experimental aspects including the individual setups, the materials used and the experimental procedures.

Chapter-III:

It deals with the mathematical analysis leading to development of models describing the multi-particle hydrodynamics pertaining to the fixed and fluidized beds and their generalization. The experimental data generated and their analysis in the form of graphs and figures are also presented.

Chapter-IV:

The correlations / theoretical models developed for voidage profile and total pressure drop / static pressure distribution of a shallow CFB are presented. The large amount of experimental data collected and their analysis also are presented. The applicability of the models / correlations developed with respect to the experimental data is also discussed in detail.

Chapter-V:

It presents a summary of overall conclusions for flow through fixed and particulate fluidized beds and flow through CFB.

Chapter-VI:

It deals with scope for further work in both the areas.

Chapters-VII, VIII & IX:

They present the nomenclature, references and the appendices respectively.

Chapter - I

LITERATURE SURVEY

I.1 Voidage, Pressure Drop and Drag in Fixed and Particulates Fluidized Beds:

Plenty of literature exists on pressure drop and drag in fixed beds, and hydrodynamics and energy dissipation of fluidized beds comprising of particulate systems. There are a good number of approaches and correlations / models describing the hydrodynamic behavior of fixed and fluidized beds separately, but all of them have limitations of voidage and particle Reynolds number ranges. Also, there have been a few attempts to generalize the equations describing the flow through fixed and fluidized beds. All most all of these equations have been based on the vast experimental data existing in the literature on fixed and particulate fluidized beds comprising of spherical particles made of glass, aluminum, ceramics and nearly spherical particles like sand and quartz. . The extensive literature survey reveals that more realistic generalization based on experiments, in particular on commercial solids, is necessary in view of the importance of wide range of porosity (voidage) and particle Reynolds number dealt with for the design of commercial importance of such liquid-solid systems. This section lastly emphasizes on the scope of present investigation.

I.2 Voidage Distribution, Pressure Gradient and Core Annulus Flow in the Riser of Circulating Fluidized Beds (CFB):

This section presents a comprehensive literature survey, dealing with several aspects of a CFB system, namely, pressure gradient, voidage distribution and core annulus flow in the riser of a CFB, accelerating or mixing length. A critical review of the literature evinces that the correlations reported for voidage and pressure drop are for large CFBs. This information for shallow CFB is really meagre. In the light of above observations, the scope of the present investigation has been emphasized.

Chapter - II

EXPERIMENTAL ASPECTS

II.1 EXPERIMENTS ON FIXED AND FLUIDIZED BEDS:

II.1.1 Experimental Set-up:

Figure II.1.1 illustrates the schematic diagram of the experimental set-up for studying flow through fixed and fluidized beds. The set-up consists of four Perspex columns of diameters 2", 3", 4" and 6". The columns are of equal length, the height of each being 1.5 m above the distributor, which is nothing but a stainless steel wire mesh firmly fixed in between two flanges. A calming section filled with raschig rings below the distributor ensures the flow of liquid devoid of pulses. Provisions exist in the set-up for measurement of flow rates of liquid by means of the calibrated rotameters. For static pressure measurements across the bed (fixed or fluidized), series of manometers filled with carbon tetrachloride have been used. The flow through fixed and fluidized beds have been carried out in 3" and 4" columns and the free fall velocity experiments on individual particles were carried out in 6" column. The 2" column has been used for the purpose of only the visual observations. The material used were non-coking coal and limestone whose details are presented in the Table II.1.1

II.1.2 Experimental Procedure:

All the necessary characteristics like size, sphericity, free fall velocity, true and bulk densities, static bed voidages have been obtained through relevant experiments conducted on the selected solids. Three static bed heights have been maintained for each column, each material and each size of the material. The total number of runs were $2 \times 3 \times 2 \times 3 = 36$. Water has been used as the fluidizing medium in all the runs. Each run has started with a particular fixed bed and the water flow through the bed has been subjected to the gradual increase. Data on pressure drop across a particular height versus volumetric flow rate of liquid have been obtained for the fixed bed. Then the increased liquid flow rates have made the bed to expand gradually after crossing the minimum fluidization point. The pressure drops and expanded bed heights versus the liquid flow rates have been

recorded for the fluidized bed. The experimental conditions are also highlighted in the Table – II.1.1

II.2 EXPERIMENTS ON CIRCULATING FLUIDIZED BED:

II.2.1 Experimental Set-up:

A schematic diagram of the experiment setup is shown in Figure II.2.1. It essentially consists of a blower (A), a riser column (F), a cyclone separator (G), a solids return leg (H), a slow bed column (I) and a solids re-circulation line (E). Other accessories include a bag filter attached to the exit end of the cyclone separator, a panel on which manometers have been mounted and orifice meters (C).

The whole setup is made up of Perspex material so as to permit visual observations of solid-air contacting patterns in the riser. The experiment were carried out with four different types of solid materials whose details are presented in Table II.2.1

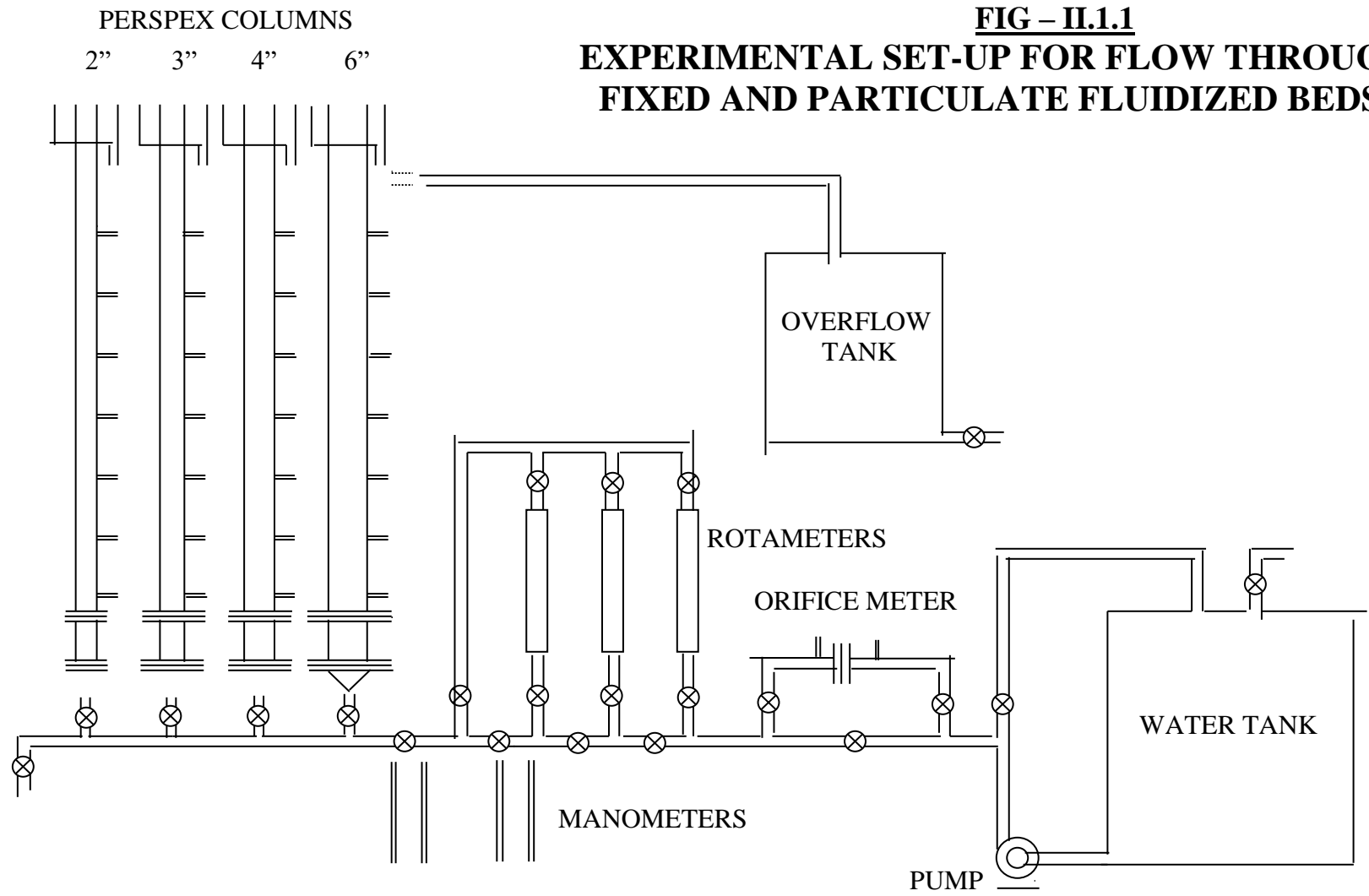
II.2.2 Experimental Procedure:

Initially static beds of solids are maintained in both fast and slow bed columns. The solids in the riser (F) are then fluidized at high velocity using air as the medium while the flow rate of air in the slow bed (I) is kept above that at minimum fluidization. The solids entrained from the riser are re-circulated into the same bed via the cyclone separator, down comer (H) and the inclined pipe (E). Visual observation shows that the fast bed consists of a dense phase region at the bottom of a dilute phase region above.

Experimented data have been collected for determining the axial voidage and static pressure distribution in the dense-dilute phases of the fast bed by varying the fast bed air velocity and the solids circulation rate. Details of its experimental conditions are also given in Table II. 2.1.

II.2.3 Tables and Figures

All the data tables and the calibration curves have been presented.



- LEGEND**
- A BLOWER
 - B AIR CONTROL VALVES
 - C ORIFICE METERS
 - D WIRE MESH DISTRIBUTORS
 - E SOLIDS TRANSFER LINE
 - F FAST BED COLUMN (RISER)
 - G CYCLONE SEPARATOR
 - H SOLIDS RETURN LEG
 - I SLOW BED COLUMN
 - J PRESSURE TAPS FOR ORIFICEMETER
 - K AIR EXIT
- P_0 to P_7 : PRESSURE TAPS
- V_0 to V_7 : QUICK - OPENING VALVES

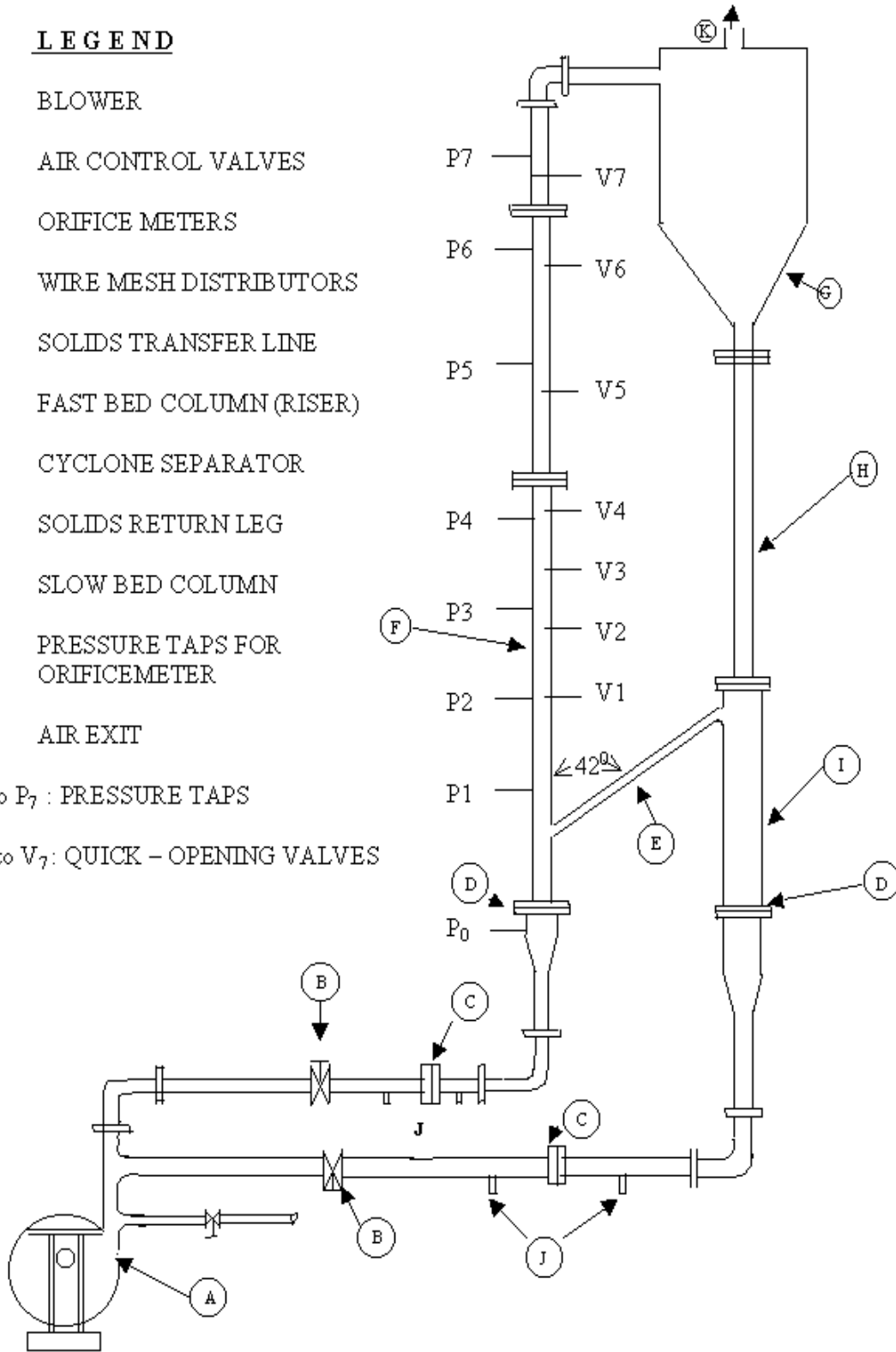


Fig-II.2.1. Schematic diagram of the experimental set-up for Hydrodynamic studies in circulating fluidized bed.

TABLE - II.1.1

Material	Size, d_p (cm)	Terminal Settling Velocity, U_t (cm/s)	Solids Density ρ_s (g/ cm³)	Static Bed Porosity
Non-coking Coal	0.1080	4.607	1.316	0.519
-do-	0.1816	6.825	1.316	0.521
-do-	0.3378	9.740	1.316	0.514
Limestone	0.1080	11.430	2.710	0.520
-do-	0.1816	16.470	2.710	0.497
-do-	0.3378	20.370	2.710	0.503

COLUMN DIAMETERS: 3" and 4"

STATIC BED HEIGHTS: 10, 15, 20 cm

MIN. FLUIDIZATION VELOCITY: 0.37 to 4.50 cm/s

TABLE - II.2.1

Material	d_p (cm)	Solid Density ρ_s (g/cm³)	Bulk Density (g/ cm³)	Static Bed Porosity	Static Bed Height, cm	Terminal Settling Velocity, cm/s
Coal -1	0.06550	1.723	0.824	0.522	15.5	396.0
Coal -2	0.078	1.685	0.753	0.553	13.0	401.0
Phospho-gypsum	0.01125	2.255	0.990	0.561	19.5	60.4
Rock – Phosphate	0.0335	2.573	1.335	0.481	15.0	308.6
Chromite- 1	0.018	4.344	2.200	0.494	12.5	360.6
Chromite- 2	0.01275	4.422	2.140	0.516	19.0	307.1

EXPERIMENTAL CONDITIONS:

U_g = Superficial Velocity of Gas in Riser, cm/s

m_s = Solids Mass Rate in Inclined Pipe, g/s

<u>Material</u>	<u>U_g</u>	<u>m_s</u>
Coal –1	469 –545	3.8 – 4.3
Coal –2	338 – 550	2.5 – 3.8
Phosphogypsum	134 – 222	6.1 – 8.7
Rock Phosphate	323 – 440	4.9 – 5.9
Chromite –1	345 – 460	9.7 – 11.4
Chromite –2	345 – 470	14.4 – 16.6

Chapter – III

FLOW THROUGH FIXED AND PARTICULATE FLUIDIZED BEDS

Two approaches have been followed – one for framing the correlations based on comparison with large amount of experimental data and correlations presented in the literature on fixed and fluidized bed separately, and the other for framing the correlations based on comparison with the experimental data generated in this work.

III.1 Generalized Correlations for Prediction of Pressure Drop and Drag Force:

A fixed bed, when upward velocity of the flow through it is increased, becomes fluidized and the porosity or voidage, the characteristic property of the bed, which is approximately 0.4 in the former case, increases in the later case and reaches unity when the bed is fully expanded, which means that the particles suspended in the fluid behave like single isolated particles. Therefore, any relation describing a fluidized bed should be extrapolated, at unit voidage, to the one applicable to an isolated particle. On the other hand, at the minimum fluidization velocity, it is assumed that flow relations for the fixed bed would be applicable to the fluidized bed. The approach here is based upon the flow under the minimum fluidization condition first and then extrapolation of the results to fixed and fluidized beds.

Under minimum fluidization condition, well known pressure drop equation is

$$\Delta P/L = (\rho_s - \rho)(1-\varepsilon)g \quad \dots\dots\dots (1)$$

This pressure drop is caused by the particles and therefore corresponds to fixed bed pressure drop.

As the bed porosity approaches unity, a particle in the bed behaves as if it were in an infinite fluid medium, and the relative velocity between the particle and the fluid approaches the superficial velocity of the fluid as the particle's net velocity is zero. The

situation is similar to the same particle settling in the infinite fluid medium with the same velocity.

When a particle is settling unhindered in an infinite fluid medium, at steady state, the drag coefficient, C_{D0} is function of the particle Reynolds number, Re_0 and the definitions are $C_{D0} = (4/3) g d_p (\rho_s - \rho) / \rho U_0^2$; and $Re_0 = d_p U_0 \rho / \mu$ (2)

For Viscous region, $C_{D0} = 24/ Re_0$, and for inertial region, $C_{D0} \sim 0.44$

For the whole region, the equation proposed by Dallavalle (1948) is

$$C_{D0} = (0.63 + 4.8 / \sqrt{Re_0})^2 \quad \dots(3)$$

From equations (1) to (3) we get

$$\Delta P/L = (3/4) (0.63 + 4.8 / \sqrt{Re_0})^2 (\rho U_0^2 / d_p)(1-\epsilon) \quad \dots (4)$$

Since we are trying to develop a coherent expression for $\Delta P/L$ that is applicable to both fluidized and fixed beds, we need to describe ΔP as a function of U_f and ϵ . That may be possible if we can establish a relationship between U_0 , U_f and ϵ that should be able to describe the bed expansion even at the minimum fluidization condition.

The pressure drop in a fluidized bed always balances the buoyant weight of the suspended particles. i.e. $\Delta P (U_f, \epsilon) = \text{constant}$; so that

$$d\Delta P = (\partial\Delta P/\partial u).du + (\partial\Delta P/\partial\epsilon).d\epsilon = 0$$

Suppose we postulate, $\Delta P (U_f, \epsilon) \propto U_f^a \epsilon^b$ (dependence on U_f^a can be justified, b is arbitrary), from the above two equations we get

$$dU_f / d\epsilon = - (b/a) U_f / \epsilon ; \quad \text{But as } \epsilon \rightarrow 1, U_f \rightarrow U_0$$

Therefore, $U_f / U_0 = \epsilon^{-b/a}$ ----- (5)

Richardson and Zaki (1954) formulated correlations for the exponent to the particle Reynolds No. i.e $Re_0 = d_p U_0 \rho / \mu$

For the present work we use the form

$$U_f / U_0 = Re / Re_0 = \varepsilon^m \quad \text{-----(6)}$$

Where $Re = d_p U_f \rho / \mu =$ Flow Reynolds No.

We can also write the same above as $Re_0 = Re \varepsilon^{-m}$ -----(7)

We know that the Archimedis number, $A_\gamma = \rho g d_p^3 (\rho_s - \rho) / \mu^2 = 3/4 C_{D0} Re_0^2 \dots$ (8)

From (3), (7) and (8), $A_\gamma = 3/4 (0.63 + 4.8 \sqrt{\varepsilon^m} / \sqrt{Re_0})^2 Re^2 \varepsilon^{-2m}$ -----(9)

Also we get from above equations (4) and (6),

$$\Delta P / L = 3/4 (0.63 + 4.8 \sqrt{\varepsilon^m} / \sqrt{Re})^2 (\rho U_f^2 / d_p) \varepsilon^{-2m} (1-\varepsilon) \quad \text{-----(10)}$$

For a bed of unit cross sectional area, $\Delta P = F_D$ where, $F_D =$ Drag force on all particles = $F_d \times N$; where $N =$ No of particles in the bed = $L (1-\varepsilon) / (\pi/6) d_p^3$ and $F_d =$ Drag force on a single particle in the bed. Therefore, $\Delta P / L = (6 (1-\varepsilon) / \pi d_p^3) F_d$ ----- (11)

And $F_d = (\pi/4) d_p^2 \cdot C_D \cdot 1/2 \rho U_f^2$ -----(12)

Which leads to $F_d = (\pi/8) (0.63 + 4.8 \sqrt{\varepsilon^m} / \sqrt{Re})^2 \varepsilon^{-2m} \rho d_p^2 U_f^2$

Or $F_d = (\pi/8) C_D Re^2 \mu^2 / \rho$ -----(13)

For all the flow conditions

$$\Delta P / L = K_1 \mu U_f (1-\varepsilon)^2 / d_p^2 \varepsilon^3 + K_2 \rho U_f^2 (1-\varepsilon) / d_p \varepsilon^3$$

At incipient point, equal with $\Delta P / L = (\rho_s - \rho)(1-\varepsilon) g$ to get

$$K_2 Re^2 + K_1 (1-\varepsilon) Re - Ar \varepsilon^3 = 0$$

Solving, $Re = (x^2 + y Ar)^{1/2} - x$ ----- (14)

Where, $x = K_1 (1-\varepsilon) / 2 K_2$ and $y = \varepsilon^3 / K_2$

But we retain x and y to be unknown; Through solving we get

$$\sqrt{Re_0} = [3.81^2 + (1.833 / \sqrt{y}) (Re^2 + 2xRe)^{1/2}]^{1/2} - 3.81 \quad \text{..... (15)}$$

Substituting either $Re \varepsilon_{mf}^{-m}$ in place of Re_0 , or $Re_0 \varepsilon_{mf}^m$ in place of Re

we get the following expressions

$$m = (-2 / \log \varepsilon_{mf}) \log [(A^2 + \sqrt{B})^{1/2} - A] \quad \text{-----(16)}$$

$$\text{where, } A = 3.81/ \sqrt{Re} ; B = (3.36/y) (1 + 2x / Re) \quad \text{-----(17)}$$

$$\text{Also we get } m = (-2 / \log \varepsilon_{mf}) \log [(P^2 + Q)^{1/2} - P] \quad \text{-----(18)}$$

$$\text{Where, } P = x / Re_0 ; Q = (y/1.333) (0.63 + 4.8 / \sqrt{Re_0})^2 \quad \text{.....(19)}$$

Just for verification of behavior of the developed expressions with respect to some information regarding fixed and fluidized bed (separate) available in the literature, ε_{mf} is selected as 0.4. For random packing of spheres its voidage varies in the range 0.36 (close) to 0.41(lose).

$$\text{Then, } m = 5.026 \log [(A^2 + \sqrt{B})^{1/2} - A] \quad \text{..... (20)}$$

$$A = 3.81/ \sqrt{Re} ; B = (3.36/y) (1 + 2x/ Re)$$

$$\text{And, } m = -2.513 \log [(P^2 + Q)^{1/2} - P] \quad \text{...(21)}$$

$$P = x/ Re_0 ; Q = (y / 1.333) (0.63 + 4.8 / \sqrt{Re_0})^2$$

$$\text{We also get from the above analysis, } C_D = (0.63 + 4.8 \sqrt{(\varepsilon^m / Re)^2} \varepsilon^{-2m} \quad \text{... (22)}$$

$$\text{And the bed friction factor, } f = (3/4) (0.63 + 4.8 \sqrt{(\varepsilon^m / Re)^2} \varepsilon^{-2m} \quad \text{... (23)}$$

The Kozeny-Carman (constant = 180) and Burke-Plummer equations are the undisputed equations applicable to fixed beds in the creeping and inertial flow regimes respectively. The pressure drop equation developed here has been matched with them and using trial and error method, the following constants have been obtained

$$x = 32.2 \text{ and } y = 0.0382$$

With the selected models of repute, the different equations developed in this work with the above given x and y values have been compared in the possible ranges of flow as well as porosity. Figures 1 and 2 for a fixed bed and figure 3 for a fluidized bed illustrate these comparisons. As can be seen, the validity of the proposed models is quite satisfactory

compared to the others in the ranges $10^{-3} \leq Re \leq 10^4$ and $0.35 \leq \varepsilon \leq 0.65$ in the case of fixed beds and $10^{-2} \leq Re \leq 2 \times 10^3$ in the case of fluidized cases.

III.2 Comparison with Experimental Data:

The developed models, with the above x and y values, have also been compared with experimental data in literature and a typical comparison is shown in figure 4. Of course, these data were of spherical or nearly spherical particles.

Wen and Yu (1966), through experiments on a number of commercial solids for data on minimum fluidization velocity gave the following constants:

$$x = 33.7 \quad \text{and} \quad y = 0.0408$$

Therefore, the experimental data have been compared with the developed models and for the systems studied and it has been distinctly observed that the matching accuracy is within $\pm 10\%$ with the following values of x and y, the models being applicable to both fixed and fluidized beds maintained in the experiments.

$$x = 25.5 \quad \text{and} \quad y = 0.0445$$

Typical comparisons in case of coal and limestone are given in figures 5 and 6.

GENERALISED EXPRESSIONS

U_r = Relative average velocity between solids particles and fluid phase

= Vectorial difference between U_f / ϵ and $U_s / (1-\epsilon)$

U_f = Superficial velocity of fluid; U_s = Superficial Velocity of solid particles

For fixed and fluidized beds (no net particle motion):

$U_r = U_f / \epsilon$ or $U_f = (\epsilon U_r)$; $Re_r = d_p U_r \rho / \mu$ and $(\epsilon Re_r) = d_p (\epsilon U_r) \rho / \mu = Re$

Generalised Spherical Multiparticle Model

$$C_D = (0.63 + 4.8 / \sqrt{Re})^2 \epsilon^{-2m}$$

Replacing Re by (ϵRe_r) and rearranging

$$C_{D\epsilon} = (0.63 + 4.8 / \sqrt{Re_\epsilon})^2$$

Where, $C_{D\epsilon}$ = Generalized Drag Coefficient = $C_D \epsilon^{2m}$

Re_ϵ = Generalised Reynolds Number = $(\epsilon Re_r) \epsilon^{-m}$

Accordingly, following are obtainable;

$$\Delta P/L = (3/4) C_{D\epsilon} Re_\epsilon^2 (\mu^2 / \rho d^3) (1-\epsilon)$$

$$C_D = C_{D\epsilon} \epsilon^{-2m}$$

$$F_D = (\pi / 8) C_{D\epsilon} Re_\epsilon^2 \mu^2 / \rho$$

$$\text{and } (\epsilon Re_r) / Re_o = (\epsilon U_r) / U_o = \epsilon^m$$

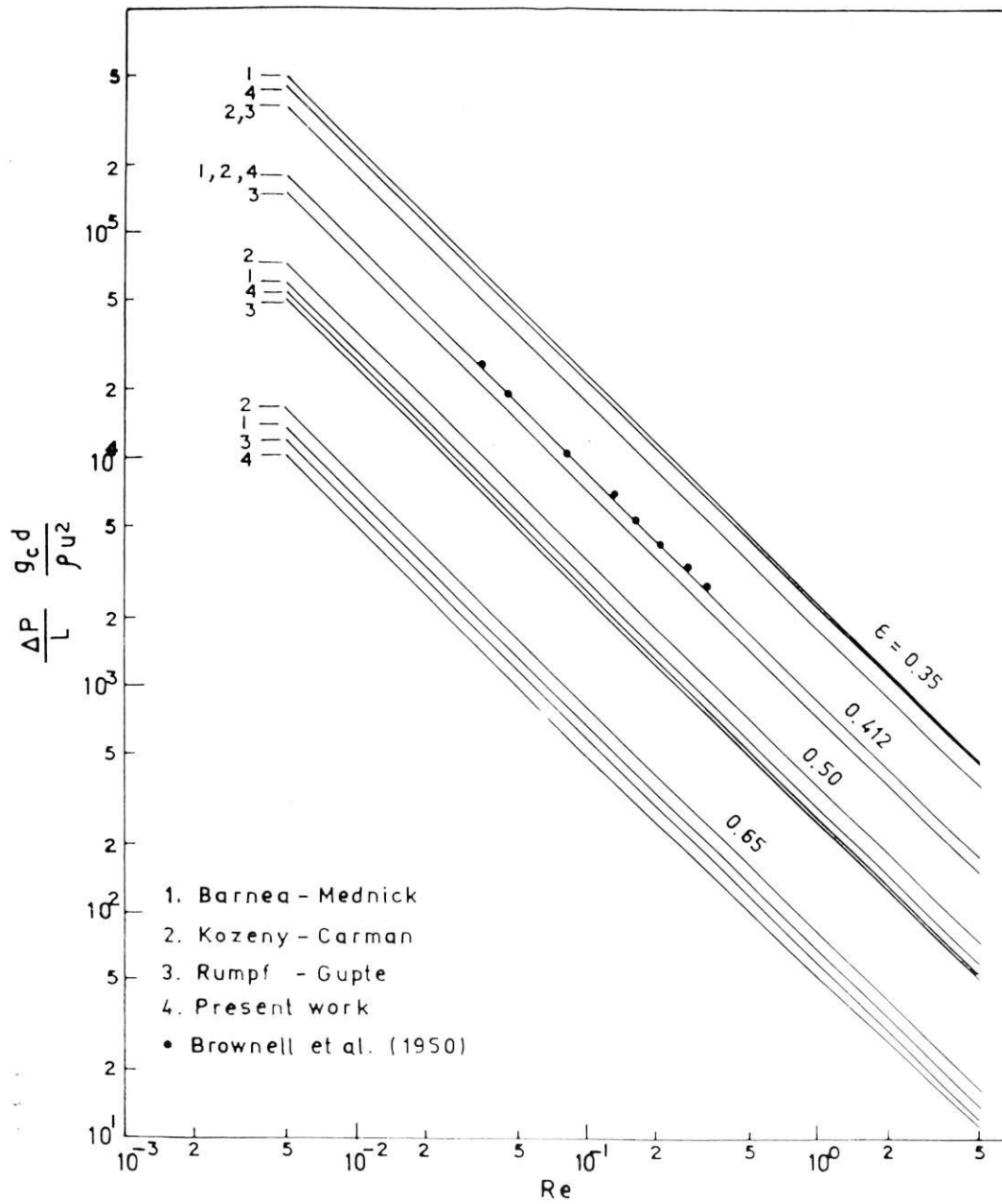


FIG.1 COMPARISON OF CORRELATIONS : CREEPING FLOW REGIME

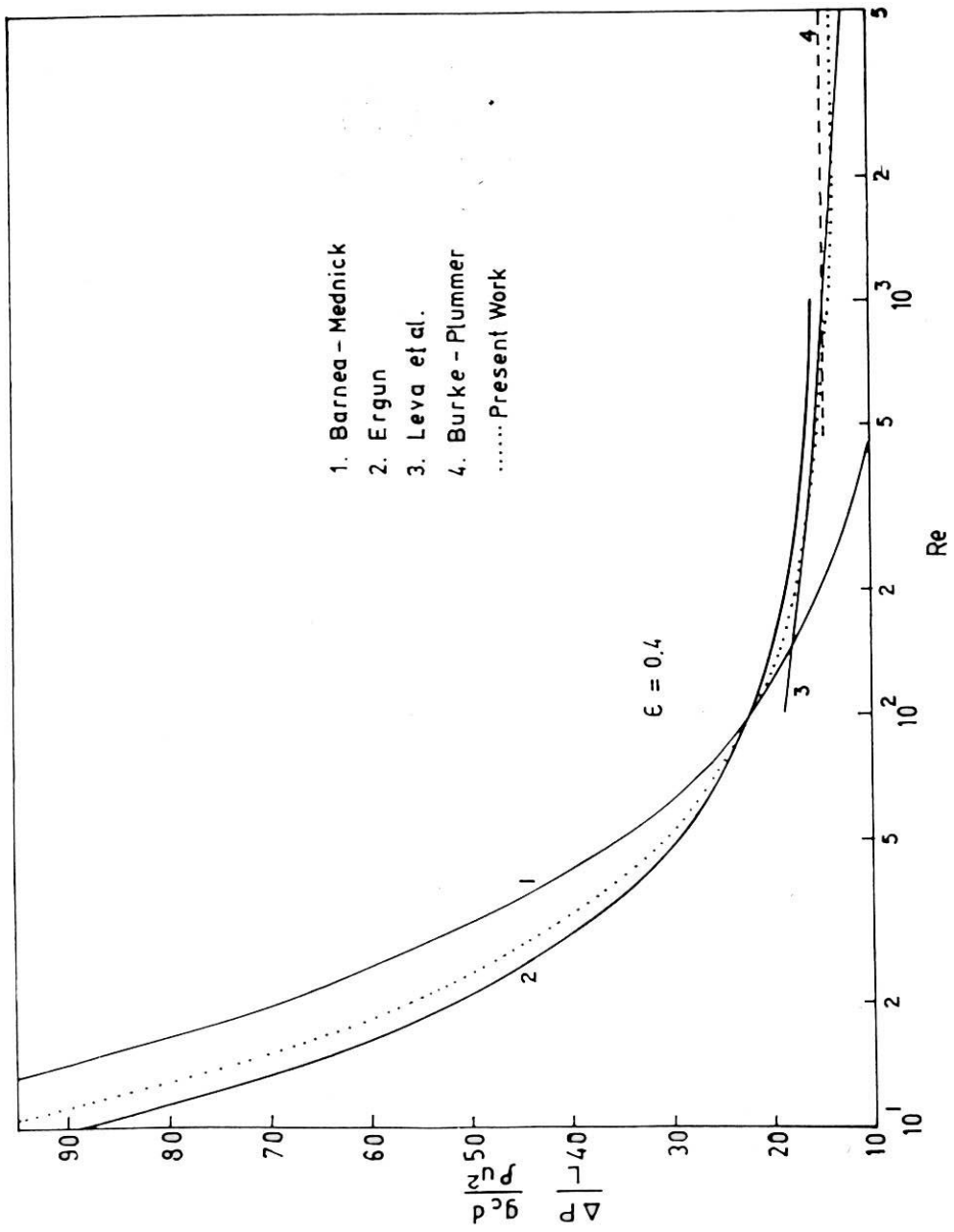


FIG. 3: COMPARISON IN THE INTERMEDIATE AND INERTIAL FLOW REGIMES.

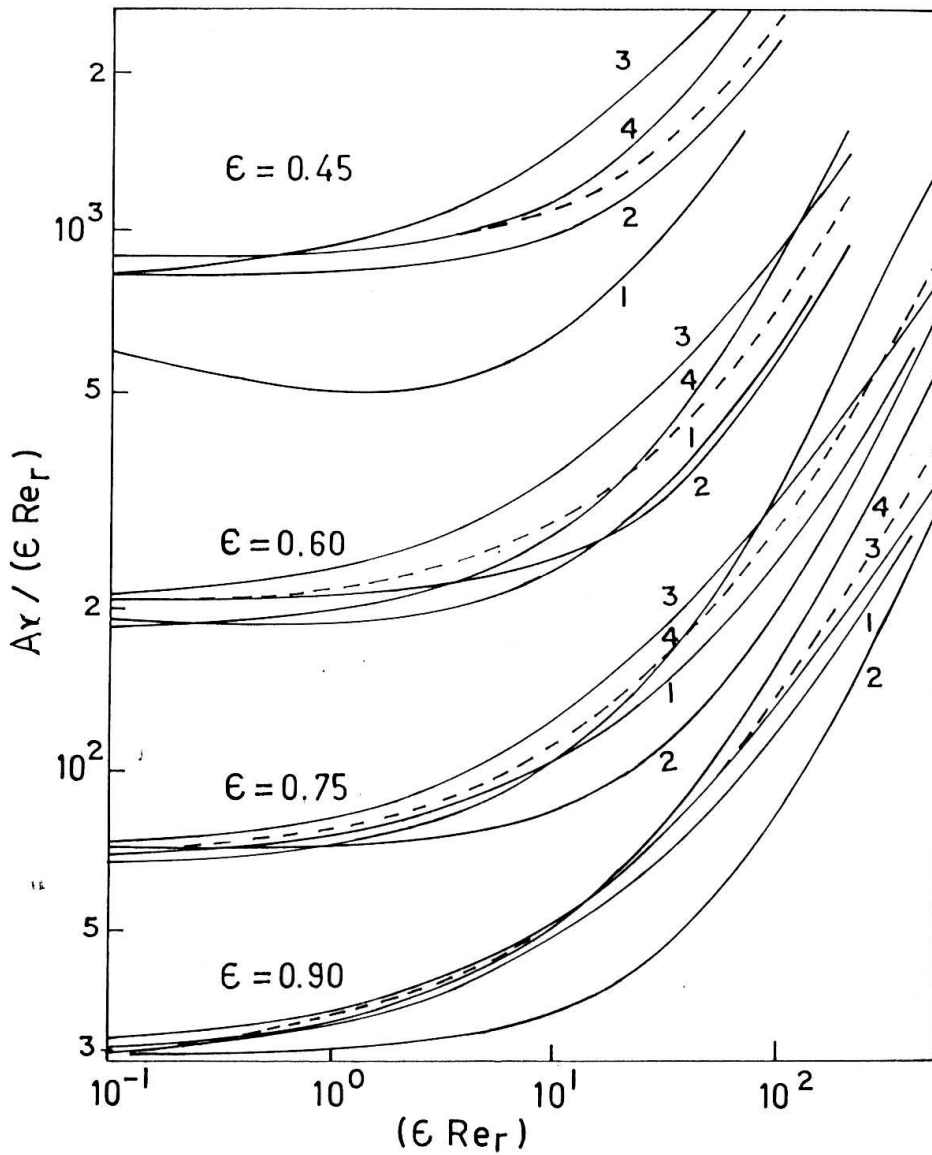


FIG. 6: VELOCITY – VOIDAGE RELATION

CURVE-1: Khan – Richardson (eq. III.53); CURVE-2: Khan – Richardson (eq. III.51); CURVE-3: Kmiec (III.50); CURVE-4: Molerus; PRESENT WORK (---): EQ. III.52

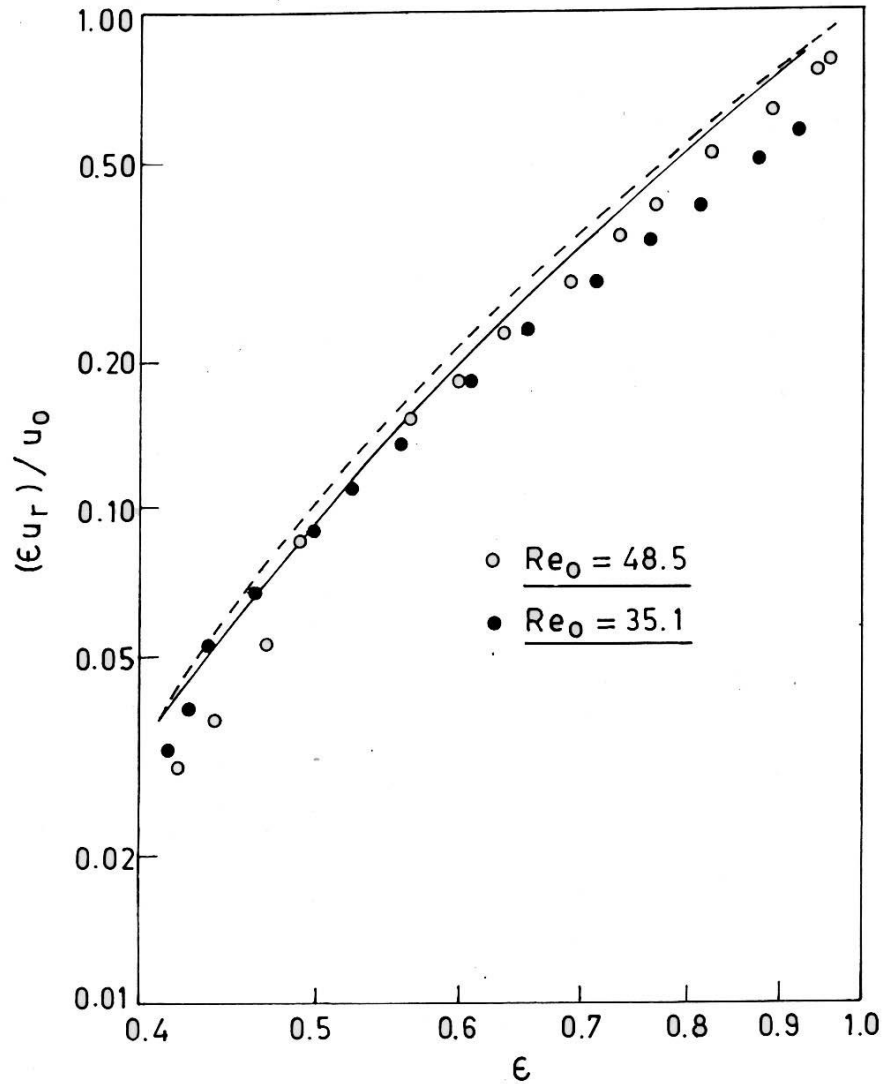


FIG. 14: PRESENT WORK vrs DATA OF WILHELM-KWAUK

- Data on Fluidization of Glass Beads by Water.
- Data on Fluidization of Sea Sand by Water.
- PRESENT WORK

FIG. 5
COAL (-3962 + 2794)
 $d_p = 0.3378 \text{ cm}$, $D=4''$, $L_s=20 \text{ cms}$

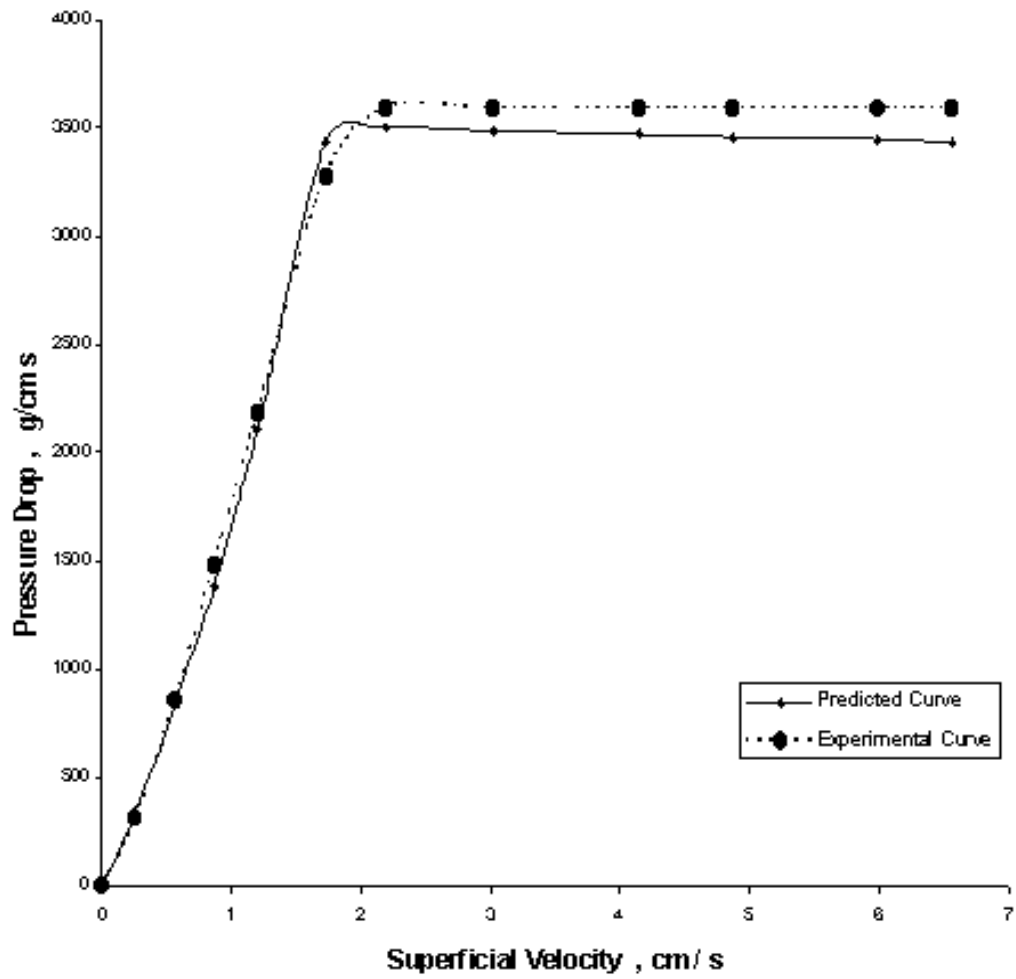
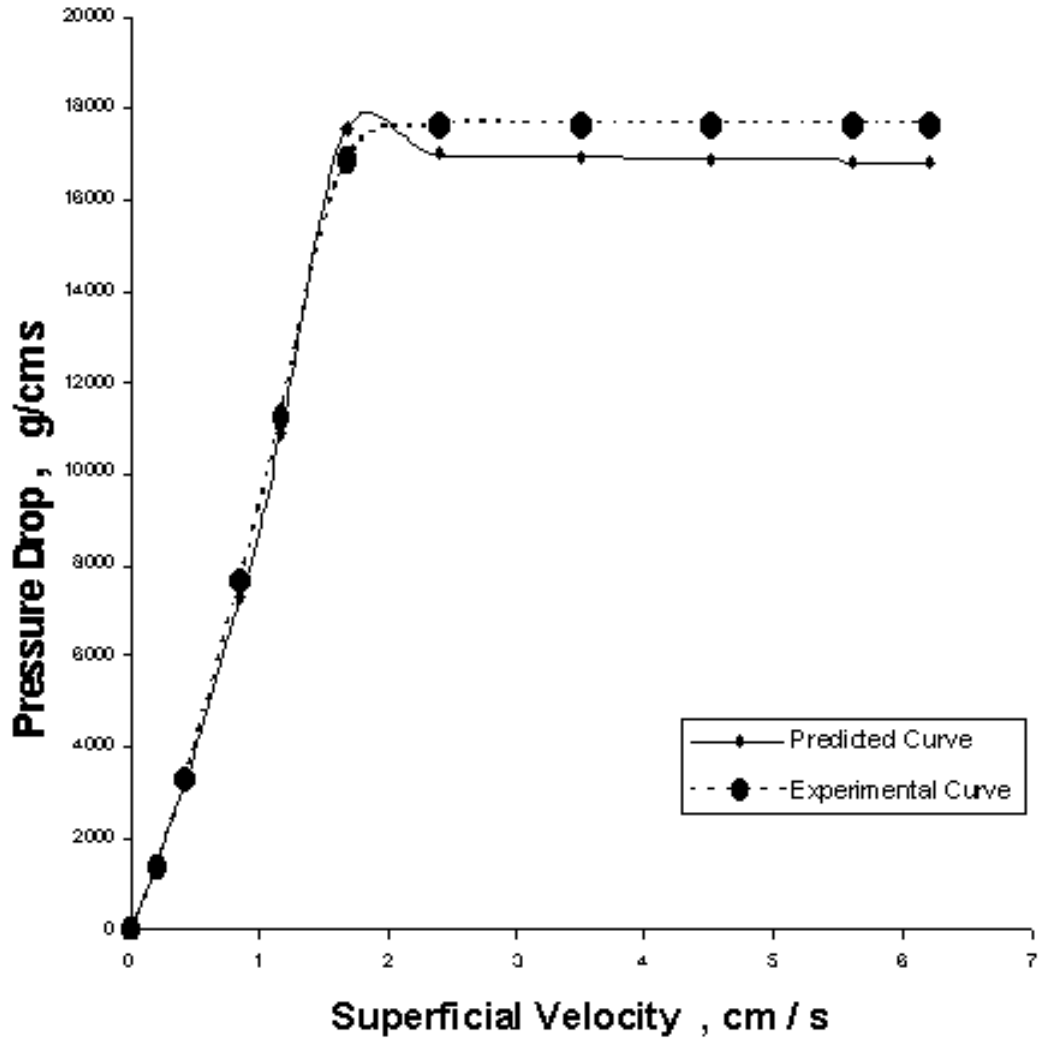


FIG - III. 6
LIMESTONE (-1168 + 991)
dp = 0.108 cm, D= 3", Ls= 20 cms



Chapter – IV

HYDRODYNAMICS OF CIRCULATING FLUIDIZED BED

IV.1 AXIAL VOIDAGE DISTRIBUTION:

The axial voidage distribution constitutes an important aspect of study in fast fluidized beds, presently being used as reactors, combustors etc. Li and Kwauk (1980) first observed that axial or longitudinal voidage distribution in the fast fluidized beds showed an “S”-shaped curve, indicating the point of inflection between the dense and dilute phase region. Measurement of voidage (or density) of CFB has been reported in many other studies (Arena et al., 1986; Hartge et al., 1988; Kato et al., 1987; and Wong et al., 1992). The correlations suggested by Kwauk et al. (1986) for an “S”-shaped voidage profile has been found to be in good agreement with some of the measured profiles. Since the point of inflection depends on the solids inventory, shape of voidage profile etc., the correlation of Kwauk et al. (1986) obviously has its limitations. Most of the workers employed risers having diameters greater than 5 cm and heights more than 2.0 m using fine particles with excessively high solids circulation rates. Consequently the findings are not strictly applicable to cases, characterizing the particle flow pattern of a shallow fluidized bed and employing coarse particles with very low solids circulation rates.

IV.1.1 Effect of Gas Velocity on Voidage:

A typical plot showing the axial voidage distribution along the riser obtained through experiments is given in FIG –IV.1.1 for the system: Noncoking Coal – Air ($d_p = 0.078$ cm). It can be observed from this figure that the nature of the curve is “S”-shaped. It has also been found that the experimental voidage increases with an increase in the riser gas velocity at a particular solids circulation rate.

IV.1.2 Effect of Particle size on Voidage:

The effect of particle size on bed voidage for the system non-coking coal-air can be seen in FIG IV.1.2. It is clear from these figures that for a particular riser gas velocity, height and solids circulation rate, voidage increases with decrease in the particle size.

The axial voidage curve shows a point of inflection (demarcation) between the dense and dilute phases. There is existence of a top region of dilute phase and a bottom region of dense phase. The point of inflection has been observed to be around 120 cm above the distributor.

An attempt was made to predict the bed voidage for the present work using the empirical correlation of Kwauk et al., (1986). However, the values obtained were found to be higher than the experimental data (the deviation being above 20%). This may be due to the fact that the CFB used in this work is a shallow one with low solids circulation rate.

IV.1.3 Correlations for Axial Voidage:

In the present work the dense and dilute phase regions were identified after determining the point of inflection by drawing tangent to each voidage distributor curve. Using the experimental void fraction data the following correlations have been proposed for the dense as well dilute phases.

DENSE PHASE:

$$\epsilon_{dn} = 0.2722 h^{0.11} R^{0.142}$$

DILUTE PHASE:

$$\epsilon_{dl} = 0.3929 h^{0.05} R^{0.126}$$

where, ϵ_{dn} is the voidage at “h” in the dense phase region,
 ϵ_{dl} is the voidage at “h” in the dilute phase region, and
h is the height above the distributor plate, cms

and,

$$R = R'_{eg} / R'_{es}$$

$$R'_{eg} = D_{fb} U_g \rho / \mu \quad \text{and} \quad R'_{es} = d_p U_s' \rho_s / \mu$$

where, U_g is the superficial gas velocity in the riser, and
 U_s' is the superficial velocity based on riser cross section of
the recirculated solids

hence,
$$R = R'_{eg} / R'_{es} = (D_{fb} / d_p) (\rho / \rho_s) (U_g / U_s')$$

The above are based on multiple regression analysis of all the experimental data.

A comparison of predicted and experimental axial voidage in the riser of the CFB for noncoking coal (both sizes and both phases) is shown in FIG IV.1.3

IV.2 Pressure Drop in Riser.

Experimental data on static pressure distribution, in case of non-coking coal (-710+600), as measured using manometers has been shown in FIG- IV.2. It can be seen from the figure that there is a smooth decrease in the static pressure with an increase in the riser height.

IV.2.1 PRESSURE DROP MODELS:

IV.2.1(a) *Pressure drop equation for two phase flow based on mechanical energy balance.*

An analysis of the total pressure for a gas-solid mixture in the fast-fluidized bed (riser) can be made from the mechanized energy balance of the gas and the solids. Thus, one may write

$$\Delta P_T = \Delta P_1 + \Delta P_2 + \Delta P_3 + \Delta P_4$$

ΔP_1 is due to the potential energy,

ΔP_2 is due to the frictional energy,

ΔP_3 is due to the kinetic energy, and

ΔP_4 is due to the momentum of the mixture

$$\Delta P_1 = \frac{(M_g + M_s) g L_T}{(Q_g + Q_s) g_c}$$

where, $M_g =$ Gas Mass Flow Rate, $v_g \varepsilon A \rho$

$M_s =$ Solids Mass Flow Rate, $v_s (1-\varepsilon) A \rho_s$

$Q_g =$ Gas Volumetric Flow Rate, $v_g \varepsilon A$

$$Q_s = \text{Solids Volumetric Flow Rate, } v_s (1-\epsilon)A$$

$$\Delta P_2 = \Delta P_f = \Delta P_{fg} + \Delta P_{fs}$$

ΔP_{fg} is due to the gas and ΔP_{fs} is due to the solids

$$\Delta P_3 = v_g^2 \rho / 2 g_c$$

$$\Delta P_4 = W V_{sl} / 2 g_c$$

The above equations can be used to predict theoretically the total pressure drop in the riser of the CFB.

IV.2.1(b) Pressure Drop Equation for Expanded Packed Bed:

An expression for the total pressure drop ΔP_T in the fast bed can also be obtained by considering the fluidized bed as an expanded packed bed as given below

$$\Delta P_T = (1 - \epsilon_{mf}) (\rho_s - \rho_g) h_{mf} g / g_c + 2 f_g v_g^2 \rho L_T / g_c D_{fb}$$

The theoretical total pressure drop calculated from the above equation is very much an underestimation with respect to the experimental data. Interestingly, the theoretical total pressure drop calculated from the Two Phase Model is very much an over estimation (deviation being more than 200%) compared to the experimental values. The reason for this deviation is due to the high values of friction factors calculated according to Yang's equation for both dense and dilute phases.

IV.2.1(c) Modified Pressure Drop Equation for Expanded Packed Bed:

In view of the above discussion, the pressure drop equation for expanded packed bed has been modified as given below. Here too the friction factor for the dense phase have been calculated using Yang's equation (Yang 1988) as such. The total pressure drop calculated from this equation is also an overestimation (deviation being more than 100%)

$$\Delta P_T = (1 - \epsilon_{mf}) (\rho_s - \rho_g) L_{mf} g / g_c + 2 f_g v_g^2 \rho L_T / g_c D_{fb} + 2 f_{dn} v_{dn}^2 \rho_s h_{dn} / g_c D_{fb}$$

All the above equations have been compared with the experimental data as shown in FIG-IV.3

IV 2.1(d) Static Pressure Distribution Approach:

In the two-phase flow literature there are a number of problems solved using two continuity and two momentum equations as follows.

Assuming the unit cross section of the riser, the common equations are as follows:

Fluid Continuity Equation:

$$\partial m_g / \partial h = 0 ; \quad m_g = \epsilon \rho v_g = \text{constant}$$

Solids Continuity Equation:

$$\partial m_s / \partial h = 0 ; \quad m_s = (1 - \epsilon) \rho_s v_s = \text{constant}$$

Fluid Momentum Equation:

$$- \partial p / \partial h = \epsilon \rho g + m_g (\partial v_g / \partial h) + f_{gs}$$

Solids Momentum Equation:

$$- \partial \sigma / \partial h = (1 - \epsilon) \rho_s g + m_s (\partial v_s / \partial h) + f_{sw}$$

p and σ are the normal pressure and stress components of the static pressure, P

Solving the above four equations with appropriate assumptions we arrive at the following

$$- \Delta P = (1 - \epsilon) \rho_s h g / g_c + \rho_s v_s^2 b \epsilon_h / g_c + 2 f_s \rho_s h v_s^2 / D_{fb} g_c$$

We need to have the values of v_s and f_s for use in equation (IV.36). Instead of using Yang's equations, an attempt has been made to derive alternatives as follows.

Irrespective of f_s , an attempt has been made to evaluate first the value of v_s , from concept of Relative Velocity Model of Gidaspow (1978). He suggested the following for vertical two-phase flow:

$$- \frac{1}{2} \frac{d}{dh} (v_g - v_s)^2 = \frac{F_s}{\rho_s} - g$$

which yields

$$(v_g/\varepsilon + v_s/(1-\varepsilon)) b \varepsilon_h / h = (18 \mu / \rho_s d_p^2) \varepsilon^{-3.65} (1+0.15 R_{es}^{0.687}) - g / (v_g - v_s)$$

where, $R_{es} = \varepsilon \rho d_p (v_g - v_s) / \mu$

However, for the solids friction factor, the choice of Yang's equation is obvious

$$f_s = 0.0126 (1 - \varepsilon)^{0.021} ((v_g - v_s)/U_t)^{0.979} / \varepsilon^3$$

But the pressure drop predictions made by this approach are quite inconsistent in case of the systems studied for this work.

IV 2.1(e) Static Pressure Distribution: Another Approach:

The three terms are designated as follows:

$$\begin{aligned} (-\Delta P)_g &= (1 - \varepsilon) \rho_s h g / g_c \\ (-\Delta P)_a &= \rho_s v_s^2 b \varepsilon_h / g_c \\ (-\Delta P)_s &= 2 f_s \rho_s h v_s^2 / D_{fb} g_c \end{aligned}$$

The following are suggested for the pressure drop equations for dense and dilute phases respectively:

DENSE PHASE:

$$(-\Delta P)_{dn} = (1 - \varepsilon_{dna}) \rho_s h_{dn} g / g_c + \rho_s v_{dn}^2 b \varepsilon_{dn} / g_c$$

DILUTE PHASE:

$$(-\Delta P)_{dl} = (1 - \varepsilon_{dla}) \rho_s h_{dl} g / g_c + 2 f_{dl} \rho_s h_{dl} v_{dl}^2 / D_{fb} g_c$$

The term for f_{dl} is same as that suggested by Yang but with a different constant as given below:

$$f_{dl} = 0.003 (1 - \varepsilon_{dla})^{0.021} ((v_{gl} - v_{dl})/U_t)^{0.979} / \varepsilon_{dla}^3$$

The predicted curve through this approach is shown in FIG-IV.3.

However, the deviation of this model from the experimental data is within $\pm 10\%$.

IV 2.1(f) *Hydrodynamic model for annular flow of solid and fluid phases to predict the experimental pressure distribution.*

Arastoopour and Gidaspow (1979) used such model to predict static pressure distribution in vertical pneumatic conveying. The flow behavior of gas and solid phases in the riser of CFB is very complex. Recent data on radial voidage profile have shown the existence of a well agitated dense bed of solids in the lower part of the riser and an upward moving dilute gas core with entrained solids in it, surrounded by down flowing thin solid film in the wall region above the dense bed (Dry, 1987; Bader et al., 1988; Bolton and Davidson, 1988 and Hartge et al., 1988).

The hydrodynamic model modified for the riser of CFB comprises the following equations:

$$\text{Fluid continuity, } d(\varepsilon \rho_g v_g) / dx = 0$$

$$\text{Solid continuity, } d((1-\varepsilon) \rho_s v_s) / dx = 0$$

Mixture momentum,

$$(1-\varepsilon) \rho_s v_s (dv_s/dx) + \varepsilon \rho_g v_g (dv_g/dx) + ((1-\varepsilon) \rho_s + \varepsilon \rho_g) = - (dp/dx) - F_w - F_{ws}$$

Pressure drop in both solid and fluid phases (annular flow model):

$$\rho_s v_s (dv_s/dx) + dP/dx = F_s - \rho_s g - F_{ws}/(1-\varepsilon)$$

Analysis of Hydrodynamic model

The equations above are the four sets of non-linear first order differential equations and have v_g , v_s , ε and P as unknowns and these four equations can be solved simultaneously using the method of Runge-Kutta.

The dimensionless form of mixture momentum equation is

$$\frac{d\tilde{P}}{d\tilde{x}} = \left\{ R \bar{v}_s S^2 \left[(1-\varepsilon_1) \frac{d\tilde{v}_s}{d\tilde{x}} + f_s \bar{v}_s L / 2D \right] + F \left[R (1-\bar{\varepsilon} \varepsilon_1) + \bar{\varepsilon} \varepsilon_1 \tilde{P} \right] + (f_g \bar{v}_g^2 \bar{P} L / 2D) \right\} / ((\varepsilon_1 - C \bar{\varepsilon} \bar{P}^2) / \bar{\varepsilon} \bar{P}^2)$$

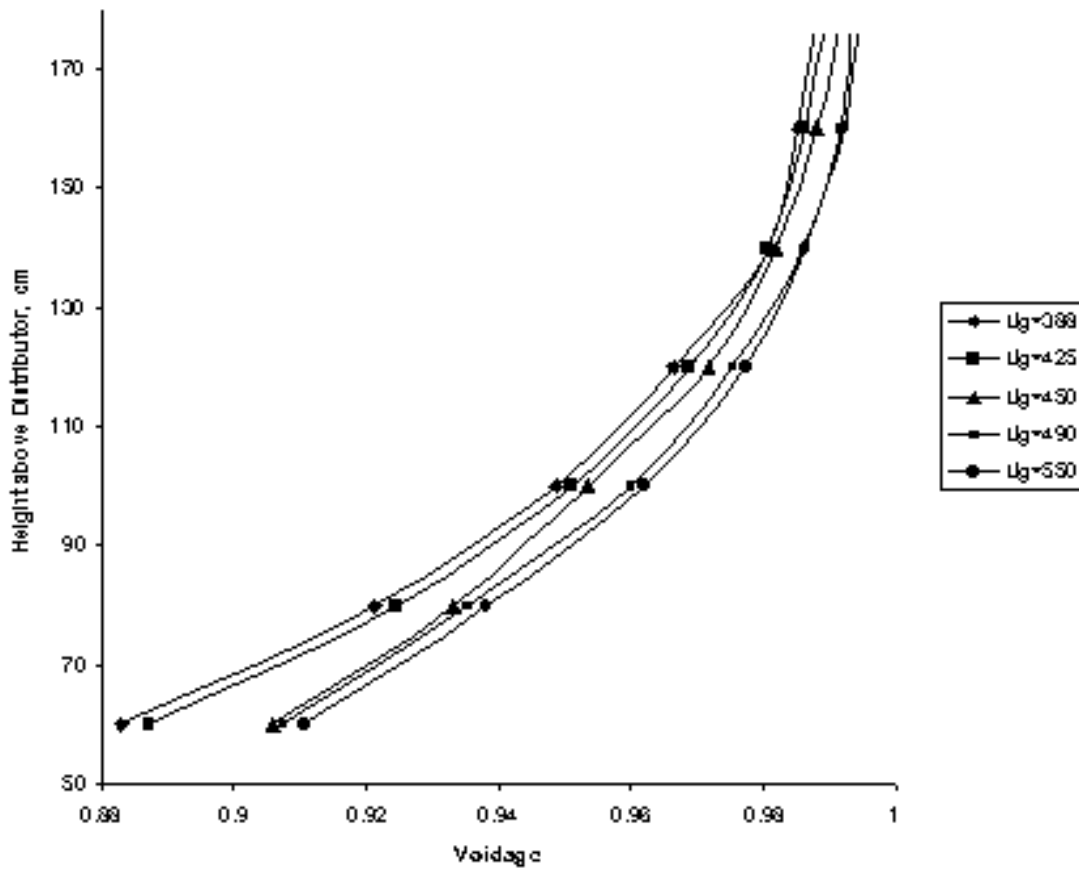
The dimensionless form of Annular Flow Model equation is

$$d\bar{v}_s / d\bar{x} = \left(\left\{ F [R (1 - \bar{\epsilon}\epsilon_1) + \bar{\epsilon}\epsilon_1 \bar{P}] - C f_s L / R S^2 2D + f_g v_g^2 \bar{P} L / 2D \right\} \right. \\ \left. + \left[G \bar{P} (v_g - S \bar{v}_s)^2 (\epsilon_1)^{0.05} (\bar{\epsilon})^{-2.65} / R \bar{v}_s S^2 - F / \bar{v}_s S^2 \right. \right. \\ \left. \left. - f_s \bar{v}_s L / 2D (1 - \bar{\epsilon}\epsilon_1) \right] \left[(\epsilon_1 - C \bar{\epsilon} \bar{P}^2) / \bar{\epsilon} \bar{P}^2 \right] \right) / \\ \left[((\epsilon_1 - C \bar{\epsilon} \bar{P}^2) / \bar{\epsilon} \bar{P}^2) + C(1 - \bar{\epsilon}) \right]$$

The simultaneous solution of the equations will give theoretical static pressure distribution, p ; solid concentration $(1 - \epsilon)$; gas velocity v_g and solids velocity v_s along the riser height. The above equations were solved to obtain the numerical results with initial condition at $\bar{x}=0$; $\bar{p}=\bar{v}_s=1$.

An attempt was made to obtain the values of solids friction factor f_s from correlation of Yang (1978) and calculate the values of total pressure drop, ΔP_T considering two-phase flow based on mechanical energy balance equation which however gave very high values of ΔP_T (deviation being more than 100%). Accordingly, the experimental values of ΔP_T have been substituted in equation for modified expanded bed for calculating total pressure drop so as to obtain the values of f_s . These values of f_s have been used in this model. The calculated values of static pressure distribution and voidage distribution using this model have been compared with experimental data of Rockphosphate-Air, as shown in figures IV.4 and IV.5. The agreement is very good and the percent deviation is only ± 5 . The theoretical gas velocity and solid velocity along the riser height calculated from annulus flow model not presented as experimental data for their verification were not obtained.

Fig IV.1.1
 NON-COKING COAL (-850+710)
 Effect of Superficial Gas Velocity on Voidage



NONCOKING COAL (-710+600)
Effect of Superficial Gas Velocity on Voidage

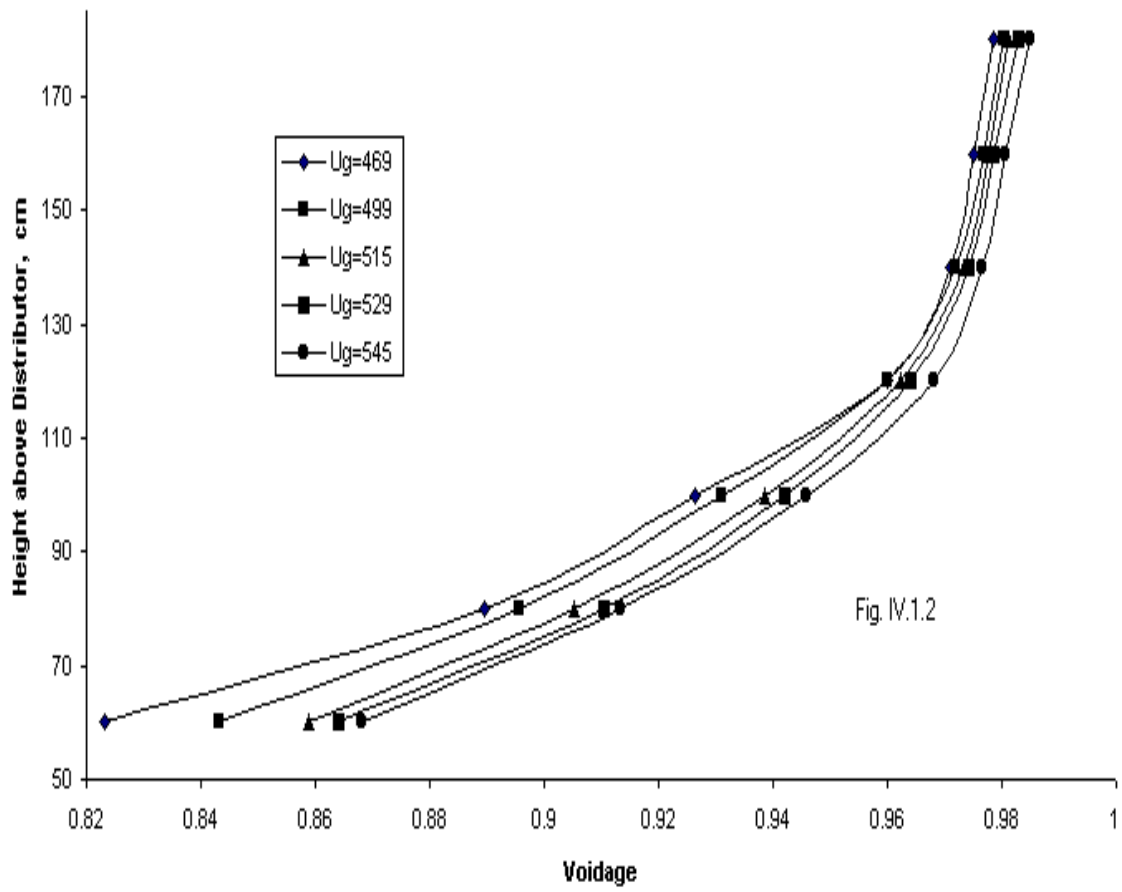


Fig. IV.1.3

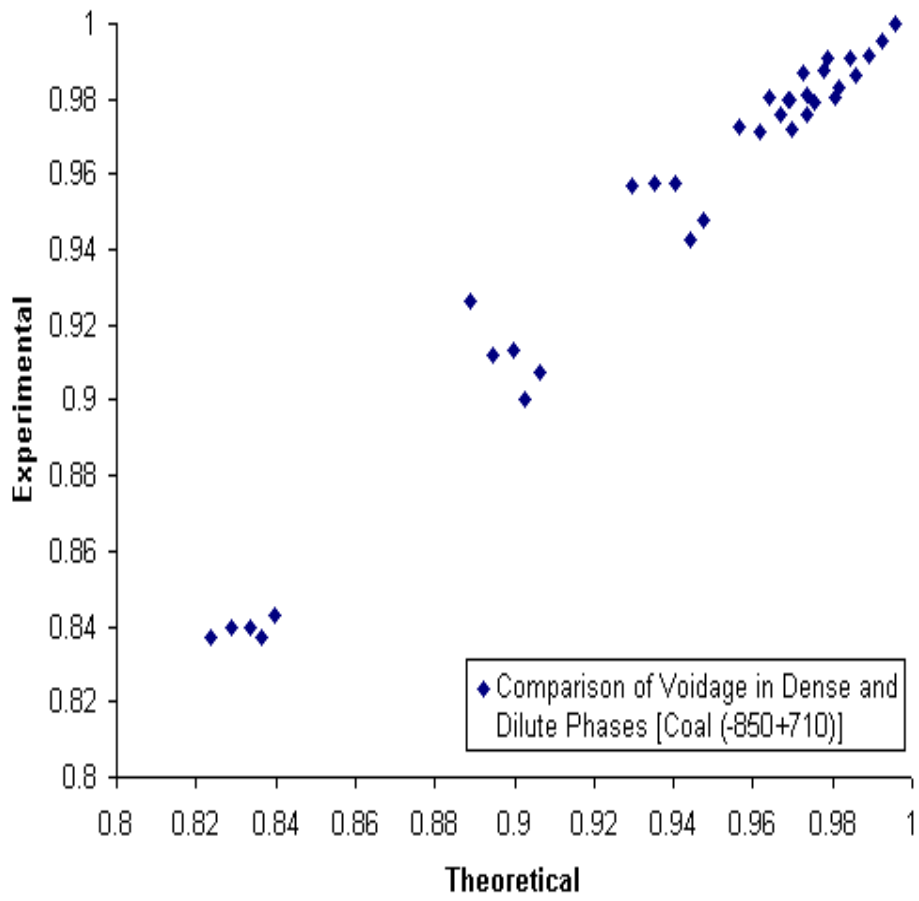


FIG. IV.2

EXPERIMENTAL STATIC PRESSURE DISTRIBUTION
Noncoking Coal (-710+600)

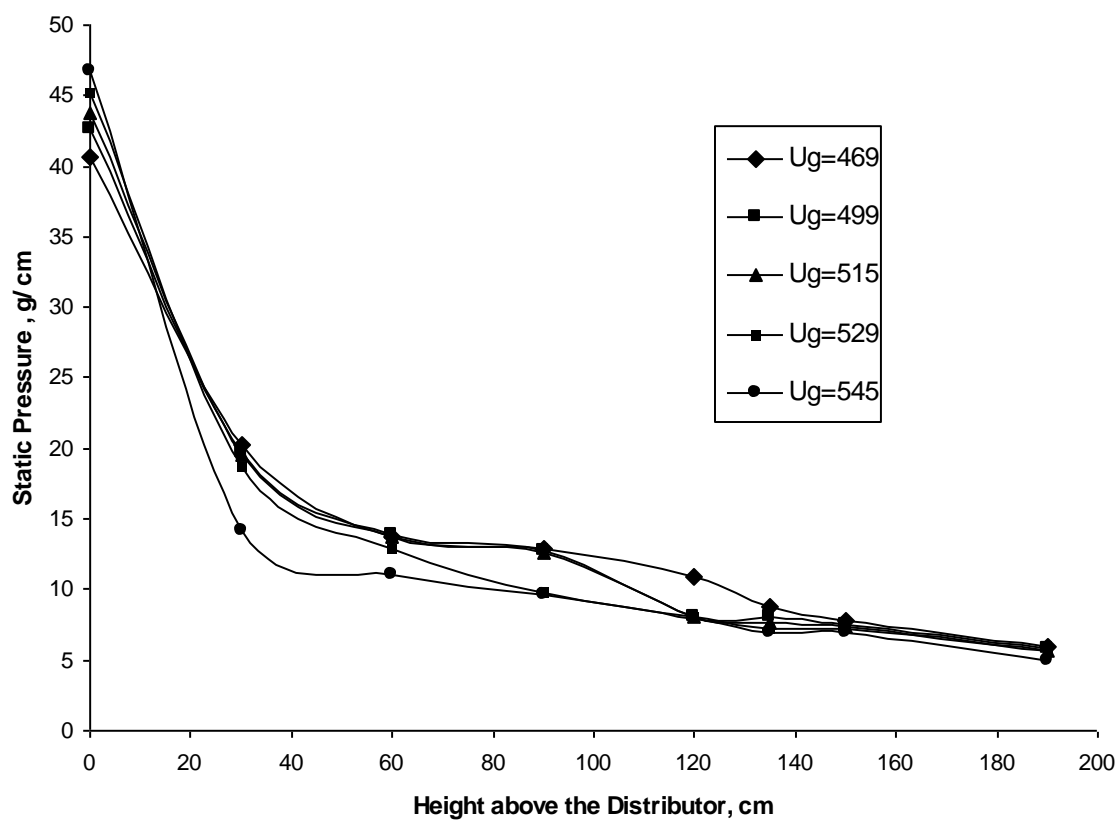


Fig. IV.3

**Comparison of Models with Experiment
COAL (-710+600) -AIR**

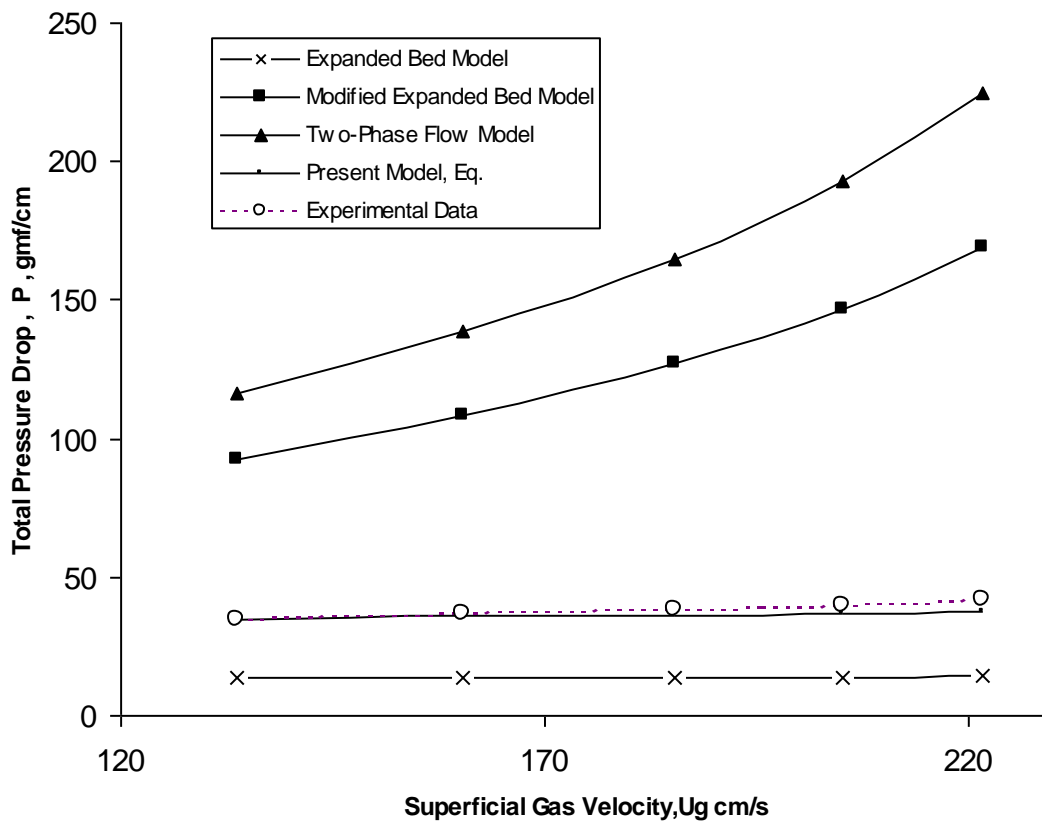


FIG. IV.4
ROCK PHOSPHATE

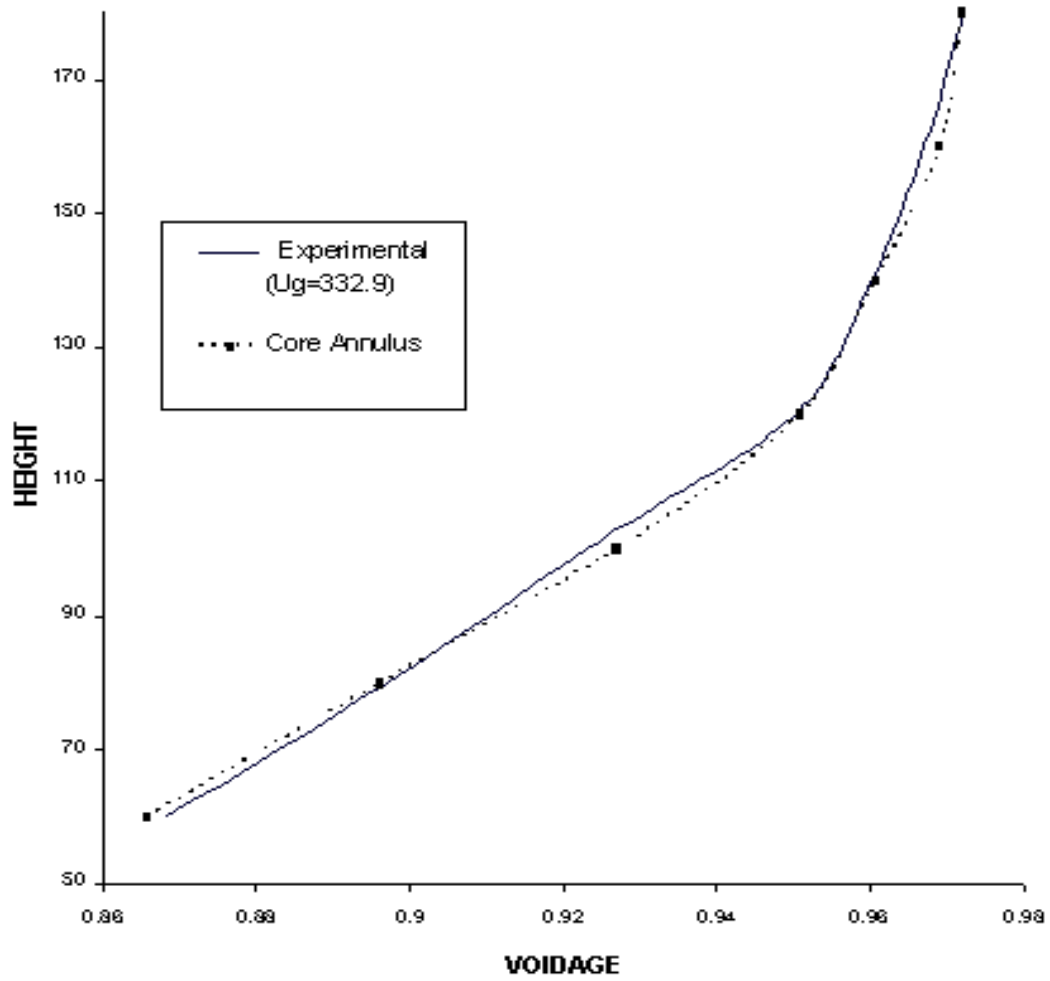
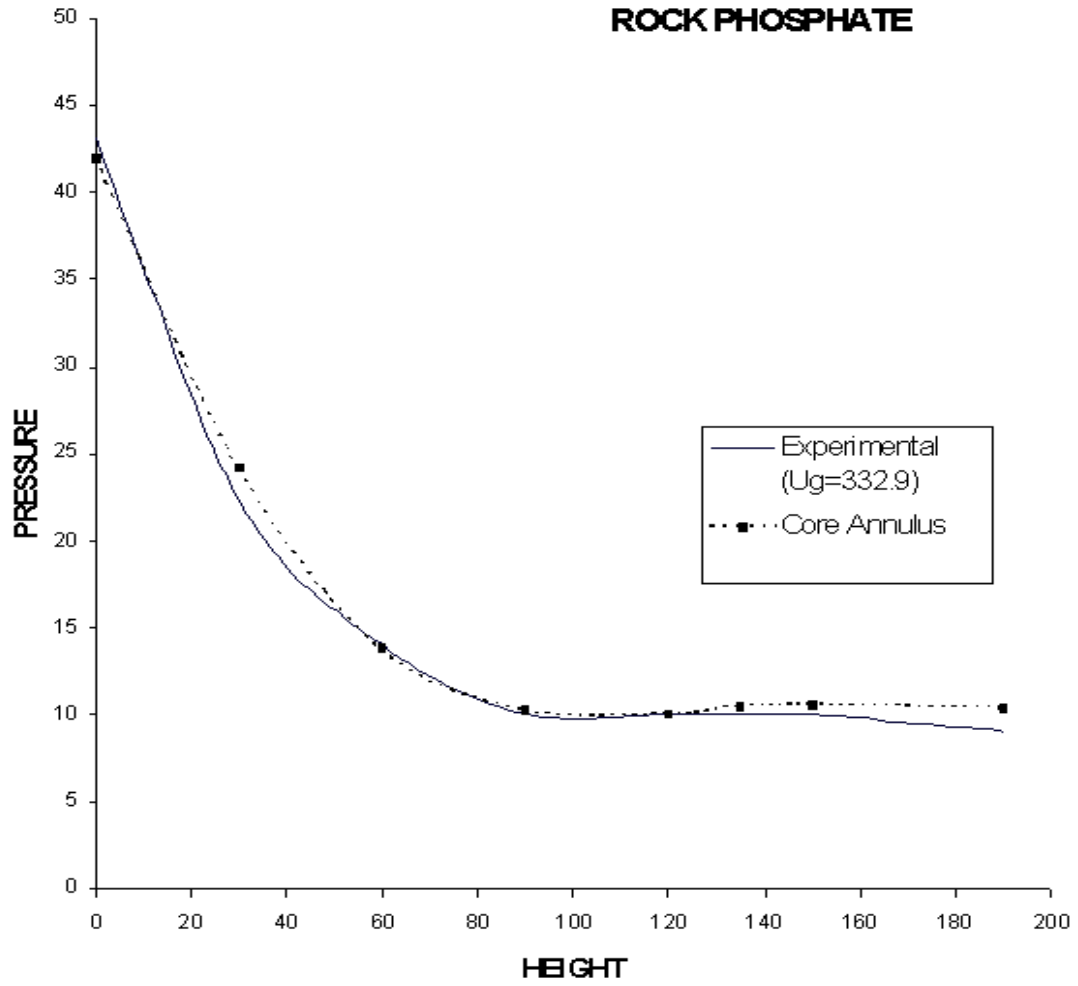


FIG IV.5
ROCK PHOSPHATE



Chapter – V

CONCLUSIONS

V.1 FLOW THROUGH PACKED AND PARTICULATE FLUIDISED BED:

- 1) The phenomenon of a single particle settling unhindered in an infinite fluid medium is analogous to that of the same particle in a bed of like particles that is in the fully expanded state due to the vertical upward flow of the same fluid and the bed porosity is approaching the unit value. Also, it is justified to assume that, at the onset of fluidization, the pressure drop equation for fixed bed is also applicable to the bed in the state of minimum fluidization. Based on such analogy and assumption, a generalized pressure drop equation that is applicable to both fixed and fluidized beds has been developed. The validity of the equation has been tested with support of large experimental data and many correlations available in the literature for fixed and fluidized beds separately. The developed correlation is found to be valid in a very wide range of porosity and particle Reynolds number unlike a few generalized models, which were developed by earlier workers.
- 2) Experimental data on non-coking coal and limestone, the solids that were never used before, have been obtained for the flow through their fixed and fluidized beds, and the validity of the semi-theoretical model developed has been established in comparison with the experimental data
- 3) An attempt has been made to frame a spherical multi-particle model that describes drag dissipation in multi-particle systems like flow through fixed and fluidized beds, sedimentation etc. A general fluid velocity term has also been defined.

V.2 FLOW THROUGH CIRCULATING FLUIDISED BED:

- 1) Experimental voidage distribution curves show point of inflection demarcating dense phase and the dilute phase in the riser. Down flow of solids along the riser wall with an upward moving concentric gas core in the dilute phase was also observed.
- 2) Empirical correlations have been proposed for predicting the axial voidage profiles in the dense and dilute phase regions for low solid circulation rates.
- 3) An attempt has been made to predict theoretical pressure drop from mechanical energy balance considerations and expanded bed analogy gave large deviations. Another approach has been made to predict total pressure drop based on modification of expanded bed equation using the corrected Yang's equation for solids friction factor. There was a good agreement between the model prediction and the experimental data, the percent deviation being ± 10 .
- 4) An attempt has been made following the method of Arastoopour and Gidaspow and considering annular flow of solid and fluid phases to predict the theoretical static pressure as well as gas voidage distribution along the riser height. The predicted static pressure distribution and voidage distribution agreed well with the experimental data, the percent deviation being ± 5 .

Chapter – VI

SCOPE FOR FURTHER WORK

VI.2 FLOW THROUGH FIXED AND PARTICULATE FLUIDISED BEDS:

The following areas in the generalized hydrodynamics of fixed and fluidized beds need further and detailed work:

- Validity of the generalized equations developed in this work is to be tested for non-spherical particles for all possible ranges of porosity and Particle Reynolds Number.
- Assuming that the generalized equations developed in this work are also applicable to other fluid-particle systems like sedimentation and hindered settling, co-current and counter-current fluidization etc., extensive experimental efforts are to be undertaken.

VI.2 FLOW THROUGH CIRCULATING FLUIDISED BED:

A number of areas in the hydrodynamics of circulating fluidized bed for gas solid systems require further studies. The following are relevant:

- (i) The voidage correlations for the dense and dilute phase regions did not include effect of height to diameter ratios of the riser. Study on such effect has immense potential in the scale-up of CFB
- (ii) An approach should be made to obtain theoretical equations for voidage distribution considering dynamics of moving bed of transport.
- (iii) Effect of different circulation rates on voidage can also be undertaken.

Chapter – VII

NOMENCLATURE

A	Cross-sectional area of the riser
A_r	Archimedis Number
C_D	Drag coefficient of a particle in bed
C_{D_o}	Drag coefficient of a particle in infinite fluid
$C_{D\epsilon}$	Generalized drag coefficient
D_{fb}	Diameter of fast bed / riser of CFB
d_p	Diameter of particle
f	Bed friction factor
F_d	Drag force on a particle in bed
F_D	Drag force on all particles in bed
f_{dn}	Solids friction factor in dense region of riser in CFB
f_{dl}	Solids friction factor in dilute region of riser in CFB
f_g	Gas friction factor in riser of CFB
f_s	Average solids friction factor in riser of CFB
F_s	Force on solids relative to gas flow in CFB
F_w	Gas phase frictional force in riser of CFB
F_{ws}	Solids phase frictional force in riser of CFB
g	Acceleration due to gravity
g_c	Newton's gravitational constant
h	Height above distributor in riser of CFB
L	Length of bed
L_T	Total height of riser in CFB
m	Constant in eq. 6 in Chapter- III
N	Number of particles in bed
ΔP	Pressure-drop across bed / riser
P	Static pressure of a particular height in riser of CFB
Re	Flow Reynolds Number in fixed / fluidized bed
Re_g	Gas Reynolds Number in riser of CFB

Re_o	Particle Reynolds Number
Re_ε	Generalized Reynolds Number
u_f	Superficial velocity of fluid in fixed / fluidized column
U_g	Superficial gas velocity in riser of CFB
u_r	Relative velocity of particles with respect to fluid
u_s	Solids superficial velocity in fixed / fluidized bed column
U_o/U_t	Terminal settling velocity of particle in infinite fluid
U_s	Superficial velocity of solids (recirculated) in riser of CFB
v_g	Actual gas velocity in riser of CFB
v_s	Actual solids velocity in riser of CFB
v_{sl}	Slip velocity between solids and gas in riser of CFB
W	Mass rate of solids in riser of CFB
x, y	Constants in eq. 5 in Chapter- III

Greek Symbols:

ε	Bed porosity in fixed / fluidized bed, or Average bed voidage in riser of CFB
ε_{dn}	Voidage at h in dense region of riser in CFB
ε_{dl}	Voidage at h in dilute region of riser in CFB
ε_{dna}	Average voidage between 0 and h in dense region of riser in CFB
ε_{dla}	Average voidage between 0 and h in dilute region of riser in CFB
ε_{mf}	Porosity or voidage under minimum fluidization condition
ρ	Fluid density
ρ_s	Solids density
μ	Fluid viscosity

Chapter – VIII

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