Abstract

Discretization of a 2-manifold surface, such as a triangle mesh, into a set of voxels is a well-known problem in digital geometry and computer graphics. A multitude of algorithms have been proposed in the past for constructing a water-tight (i.e., gap-free) voxel model for a triangle mesh. However, very few of them have addressed the problem with construction of *thinnest voxelization* and related issues. This thesis investigates this problem and provides a mathematical formulation for thinnest voxlization using the concepts of *Diophantine cover* and bijective projection for every individual triangle of a mesh. New characterization and interesting properties of the thinnest voxel set have been determined, and an efficient algorithm is proposed for voxelizing the triangle mesh. The algorithm can be executed both in CPU and GPU environments.

Uniform size and shape of voxels and their well-structured neighborhood provide enormous computational advantage for geometric and topological analysis compared to continuous (e.g., triangle mesh) representation. However, for voxel-based analysis, the voxelized model must contain enough details of the underlying object. This requires estimation of appropriate resolution of voxelization for retaining important geometric and topological properties of the object. It is directly related to the clearance between different parts of the object while presented as a triangle mesh or as a voxelized model. The clearance between different parts such as tunnels/handles, protrusions, and surface patches in the triangle mesh is important for determining necessary separation in their discrete counterpart (i.e., voxel set). We have shown how this can be characterized by using a novel measure called *inter-simplex Chebyshev distance* (ICD). An efficient algorithm is proposed for ICD computation, which provides the necessary resolution of voxelization. Our next task is developing effective and efficient algorithmic tools for performing topological analysis of a voxelized object. In order to achieve that, the concept of *discrete level set* (DLS) is introduced, that can be constructed on a voxelized surface with the assurance of certain topological properties. This eventually aids in construction of *discrete geodesic Reeb graph* (DGRG) on the voxelized object, for topological analysis. Under various transformations like rotation and topology-constrained anisotropic deformation, a DGRG remains invariant to typical topological features like loops or cycles, which eventually helps in identifying topological features, such as 'handles', in the underlying object. Experiments on different datasets exhibit promising results on

the practical usefulness of DLS and DGRG towards extraction of high-level topological features of arbitrary voxel sets.

In order to show the merit of the voxel-based computing with the model obtained by our algorithm, this thesis also includes two applications—bimodal carving on voxelized surface and semantic segmentation of temples. For the first one, a novel technique is proposed for performing pattern-guided carving on an orientable 2-manifold surface. Its novelty lies in processing the voxelized surface using certain theories and deductions of digital geometry. The proposed carving pipeline is bimodal in nature in the sense that it can generate both negative and positive carvings, as needed, by *carve in* and *carve out* alongside the specified pattern. This is efficiently doable on a voxelized surface when it is thinnest in a digital-geometric sense. For large and complex patterns, the voxelized surface is partitioned into multiple *functional components* and a local optimization is used in order to achieve a realistic carving. Necessary theoretical foundations, explanations, implementation details, and experimental results have been furnished to adjudge the merit of the proposed technique. For the second application, a novel segmentation technique is proposed for semantic analysis of Asian temples. Mereotopological concepts such as parthood, overlap, and connection are imbibed in the segmentation algorithm for determination of various mereological relations among the segments and their hierarchies. All the computational operations are performed in the digital space using the voxelized models of the temples. Different architectural elements of a temple are finally identified by semantic analysis of the segments. Experimental results on distinct temples from Asian subcontinent indicate the efficiency and elegance of the proposed technique.