Abstract

The present thesis deals with some problems related to non-Euclidean geometries associated with the Lie groups $\mathrm{SL}(2;\mathbb{R})$ and $\mathrm{SL}(3;\mathbb{R})$ using Erlangen program. At first, we studied all the homogeneous spaces of $\mathrm{SL}(2;\mathbb{R})$ generated by the zero, one, two, and three-dimensional subgroups. We define a transitive group action, which is the Möbius transformation in this case. The two-dimensional homogeneous spaces are $\mathrm{SL}(2;\mathbb{R})/A$, $\mathrm{SL}(2;\mathbb{R})/N$ and $\mathrm{SL}(2;\mathbb{R})/K$, which are isomorphic to the upper half plane of double, dual and complex numbers. We denote the spaces as hyperbolic, parabolic and elliptic respectively, where $A,\ N$ and K are one-parameter subgroups of $\mathrm{SL}(2;\mathbb{R})$ (up to conjugacy).

From the two-dimensional continuous subgroup F of $\mathrm{SL}(2;\mathbb{R})$, we generate an one-dimensional homogeneous space $\mathrm{SL}(2;\mathbb{R})/F$, which is isomorphic to the extended real line. The action in this case is also the Möbius transformation.

Further, we have studied the action of the one-parameter subgroups in the two-dimensional homogeneous spaces. Also, we have calculated the orbits of the subgroups ($A,\ N$ and K) under the Möbius transformation. The circles, parabolas with the vertical axis of symmetry and rectangular hyperbolas of one branch are invariant in the elliptic, parabolic and hyperbolic spaces. We have extended the previously obtained result which states that, these geometric figures are only invariant when they lie in the upper half plane. To do this, we have considered another two subgroups, (namely, A' and N', both of one-dimension) and two new decompositions of $\mathrm{SL}(2;\mathbb{R})$.

Finally, we define the projective action of $\mathrm{SL}(3;\mathbb{R})$ on $\mathbb{R}^2$. We have considered all the one-parameter subgroups (up to conjugacy) of $\mathrm{SL}(3;\mathbb{R})$ and constructed their orbits in two-dimensional homogeneous space by defining the projective action. We obtain the underlying geometry under this action of $\mathrm{SL}(3;\mathbb{R})$ by finding the corresponding invariant projective properties. We have also discussed the n-transitivity of the action of $\mathrm{SL}(3;\mathbb{R})$.

Keywords: Erlangen Program, Lie group, $\mathrm{SL}(2;\mathbb{R})$, $\mathrm{SL}(3;\mathbb{R})$, Möbius transformation, homogeneous spaces, elliptic space, parabolic space, hyperbolic space, invariants, transitivity, projective action, cross-ratio, cycles, fixed points, group action, isotropy subgroup, derived representation, curvature, vector field, orbit, Iwasawa decomposition, real projective plane, homogeneous coordinate.