ABSTRACT

This thesis addresses some interesting problems on limit theorems for some dependent models and the construction of new bivariate mixtures of discrete distributions for correlated random variables. In Chapters $2-5$, we propose four new conditional models and derive the limit theorems such as the laws of large numbers, the central limit theorems and the law of iterated logarithms. In Chapters $6-7$ of this thesis, we develop two new bivariate mixture of discrete distributions. For some practical interest, the applicability of the proposed distributions are also illustrated with real data sets.

In **Chapter 1**, we briefly introduce some remarkable results and information about the limit theorems such as the strong law of large numbers, the central limit theorem and the law of iterated logarithm under various independent and dependent structures. Some of the essential results are given and introduced some appropriate notations. Moreover, a detail discussion about the construction of a new distribution is elaborated.

In **Chapter 2**, a new class of dependent and non-identical Bernoulli random variables is defined. Here the probability of success at a given trial is a function of the number of successes and probabilities of successes in the previous trials. The moment structure for this model is derived. Further, the strong law of large numbers, the central limit theorem and the law of iterated logarithm are established under a condition that the success probabilities be monotone. Simulations are carried out to demonstrate the law of large numbers and the central limit theorem.

Next, a new conditional mean model is proposed which has an application in scan statistics processes. This is the main goal in **Chapter 3**. We investigate the problem of dependent multiple scan statistic process. The limit theorems for dependent multiple scan statistic are established. The simulation study for the law of large numbers and the central limit theorem along with its goodness of fit test are also presented.

In the previous chapters, we deal with previous all-sum dependent models. In **Chapter 4**, we present a new concept for dependent and identically distributed Bernoulli random variables. A new class of previous $k(k \geq 1)$ sum dependent model is proposed. The model is an extension of the Markov chain models and others. A martingale difference sequence with its upper and lower bounds is defined to establish the limit theorems. Moreover, we have shown that the random variables tend to be uncorrelated as the distance of indices of random variables tends to infinity. The law of large numbers and the central limit theorem are illustrated through simulations. The Chi-square goodness of fit tests are performed at the end to validate the central limit theorem.

Dependency in two dimensional arrays of the sequence of correlated Bernoulli ran-

dom variables which satisfies a Markov chain criterion, is introduced in **Chapter 5**. The limit theorems for a double sum of row wise independent random variables are derived. The strong law of large numbers and the central limit theorem are validated through simulations. The Chi-square goodness of fit tests are carried out at the end to validate the central limit theorem for double sums of correlated Bernoulli random variables.

In **Chapter 6**, we introduce a new bivariate mixture of negative binomial distribution. The mixture model is obtained with two conditionally independent negative binomial distributions and bivariate beta distributions using the Bayesian approach. Some characteristics of the propose bivariate distribution are also established. The applications of the bivariate distribution are illustrated using real count data sets.

In **Chapter 7**, a new zero inflated bivariate discrete distribution is introduced. This work is an extension of previous chapter to the zero inflated case. Various properties such as moments, the probability generating function, conditional probabilities, conditional expectations are also derived. To validate the proposed zero inflated bivariate distribution, we use two real count data sets with the occurrence of excess number of zeros.

In **Chapter 8**, we introduce some future scope of research in the direction of limiting distributions and construction of new distributions.

Keywords: Bivariate beta distribution, Bivariate discrete distribution, Central limit theorems, Dependent Bernoulli random variables, Generalized binomial distribution, Generalized hypergeometric function, Law of iterated logarithm, Markov chain model, Martingale, Maximum likelihood estimation, Moving windows, Multiple scan statistic, Multivariate discrete distributions, Poisson binomial distribution, Short and long range dependent, Strong law of large numbers, Zero inflation.