Abstract

Many graph colorings are motivated by *frequency assignment problem* which is a problem of assigning frequencies to transmitters in an optimal way and with no interference. Interference can occur if transmitters located sufficiently close to each other receive close frequencies. At the same time the demand for frequencies is outstripping the pace of technological change in expanding the usable radio spectrum. Frequency assignment problems can be modeled graph theoretically by taking stations as vertices of the graph and two vertices are adjacent if interference (up to some extent which is fixed for a given set of stations) is possible between them. Radio k-coloring has a connection with the frequency assignment problem and mathematically it is defined as follows. For any simple connected graph G with diameter d, and an integer k, $1 \leq k \leq d$, a radio k-coloring is an assignment f of positive integers to the vertices of G such that $|f(u) - f(v)| \geq 1 + k - d(u, v)$ for any two distinct vertices u and v of G. The maximum positive integer assigned by f to some vertex of G is called the span of f. The minimum of spans of all possible radio k-colorings of G is called the radio k-chromatic number, denoted by $rc_k(G)$, of G. If $k = d$, then $rc_k(G)$ is called the radio number of G. Since finding radio k-chromatic number is highly nontrivial, it is known for very few graphs and for some particular values of k only.

In this thesis, we investigate radio k-chromatic numbers of paths, hypercubes, some classes of m-distant trees, some classes of generalized Petersen graphs and circulant graphs. We determine $rc_{n-3}(P_n)$ (nearly antipodal number of P_n) and for n odd we find radio $(n-4)$-chromatic number of P_n. We improve the upper bound of $rc_k(P_n)$ for several values of k, and improve the lower bound of the same for any odd integer k. We also study a conjecture on $rc_k(P_n)$ and make a sharper conjecture than this. We find radio number of Q_n and improve the existing lower bound of $rc_k(Q_n)$ for other values of k, and give a lower bound for radio k-chromatic number of powers of Q_n. We determine radio numbers of some classes of caterpillars and extend the technique to find the radio numbers of some m-distant trees. We give a lower bound for radio k-chromatic number of

an arbitrary graph G in terms of k, number of vertices n and a positive integer M satisfying $d(u,v) + d(v,w) + d(w,u) \leq M$, for any three vertices u, v and w in $V(G)$. Smaller M-value of the graph gives better lower bound. Therefore we try to find the smallest M-values of the graphs. We improve the existing lower bounds of $rc_k(G)$ for several graphs by finding their smaller M-values.

Key Words: Frequency assignment problem; radio k-coloring; span; radio k-chromatic number; radio number; antipodal number; nearly antipodal number; paths; hypercubes; m-distant trees; caterpillar; generalized petersen graphs; circulant graphs.