Abstract

Researchers have introduced several matrices associated with a graph to model some real life problems and study graph theoretic properties. In this thesis our main objective is to study the distance matrix and distance signless Laplacian matrix associated with a graph. The *distance matrix* of a simple connected graph G is $D(G) = (d_{ij})$, where d_{ij} is the distance between ith and jth vertices of G. The *distance signless Laplacian matrix* of graph G is $D_Q(G) = D(G)+Tr(G)$, where $Tr(G)$ is a diagonal matrix whose ith diagonal entry is the transmission of the ith vertex in G.

We introduce a new class of graphs, namely, k-partitioned transmission regular graphs, and find a quotient matrix of the distance matrix (respectively distance signless Laplacian matrix) associated with a graph in this class to obtain the distance spectral radius (respectively distance signless Laplacian spectral radius). Applying this technique we find distance and distance signless Laplacian spectral radius of the wheel graph W_n, a subclass of spider graphs, and generalized Petersen graphs $P(n, k)$, $k = 2, 3$.

To determine the adjacency spectrum of a distance regular graph we have a tridiagonal matrix whose entries are the intersection numbers of the graph. In this thesis we find a similar kind of quotient matrix corresponding to a distance regular graph to find its distance spectrum. Then we prove that distance regular graphs with diameter d have at most $d+1$ distinct distance eigenvalues. We also prove that distance regular graphs satisfying $b_i = c_{d-1}$ have at the most $\lceil \frac{d}{2} \rceil + 2$ distinct distance eigenvalues. Applying these results we find the full distance spectrum of some distance regular graphs including the well known Johnson graphs. Finally we disprove a question asked by Lin *et al.* [Linear Algebra Appl., 439 (2013), 1662-1669]. In the same paper, the authors have asked for existence of graphs other than strongly regular graphs and some complete k-partite graphs having exactly three distinct distance eigenvalues. Here we construct some classes of graphs with arbitrary diameter which satisfy this property. For each $k \in \{4, 5, \cdots, 11\}$, families of graphs that contain graphs of each diameter greater than $k - 1$ are constructed with the property that the distance matrix of each graph in the families has exactly k distinct eigenvalues. While making these constructions we have found the full distance spectrum of square of even cycles, square of hypercubes, corona of a transmission regular graph with K_2, and strong product of an arbitrary graph with K_n.

We give some upper and lower bounds for the spectral radius of a nonnegative matrix. Applying this result we find upper and lower bounds for the distance and distance signless Laplacian spectral radius of graphs and obtain the extremal graphs for these bounds. Also we give upper bounds for the modulus of all distance (respectively distance signless Laplacian) eigenvalues other than the distance (respectively distance signless Laplacian) spectral radius of graphs. These bounds are probably first of their kind as we do not find in the literature any bound for these eigenvalues. Then for some classes of graphs we show that all distance (respectively

distance signless Laplacian) eigenvalues other than the distance (respectively distance signless Laplacian) spectral radius lie in the smallest Geršgorin disc of the distance (respectively distance signless Laplacian) matrix.

A positive eigenvector corresponding to the largest eigenvalue of a symmetric, non-negative and irreducible matrix is known as *principal eigenvector* of the matrix. For a real number p, $1 \leq p < \infty$, a principal eigenvector $(y_1, y_2, \cdots, y_n)^T$ is said to be *p-norm normalized* if $(\sum_{i=1}^{n} y_i^p)^{\frac{1}{p}} = 1$. We find some upper and lower bounds of the maximal and minimal entries of the p-norm normalized principal eigenvector for the distance matrix and distance signless Laplacian matrix of a graph and show that transmission regular graphs are extremal for all these bounds. We also compare these bounds with the existing ones.

Keywords: Distance matrix, Distance eigenvalue, Distance spectral radius, Distance spectrum, Transmission regular graphs, k-partitioned transmission regular graphs, Generalized Petersen graphs, Distance regular graphs, Quotient matrix, Johnson graphs, Power of graphs, Hypercubes, p-norm normalized principal eigenvector, Geršgorin disc, Distance signless Laplacian matrix.