Chapter 1

Introduction

1.1 Flow Separation

The influence of friction between the surface and the viscous fluid adjacent to the surface act to create a frictional force which retards the relative motion. This has an effect on both the surface and the fluid. The surface experiences a 'tugging' force in the direction of the flow, tangential to it. This tangential force per unit area is defined as the shear stress. The fluid adjacent to the surface feels a retarding force due to an equal and opposite reaction. The frictional force reduces the fluid velocity to zero right at the surface. This is called the no-slip condition in the viscous flow.

In addition to the generation of shear stress, friction also plays another role in the flow over the body. Assume that the flow over the surface produces an increasing pressure distribution in the flow direction. The velocity of the fluid is already retarded by the effect of friction; in addition, it must work its way along the flow against an increasing pressure, which tends to further reduce the velocity. Consequently, the fluid velocity decreases as it moves downstream along the surface and comes to a stop. Under the influence of the adverse gradient, the flow direction reverses and starts moving back upstream. This phenomenon is known as flow separation. The influence of friction not only causes the generation of shear stress, it can also cause the flow over a body to separate from the surface.

In a steady flow the body surface is a streamline. Although the velocity is zero at the surface, it has a non-zero value at an arbitrarily small distance from the wall. When particles approach each other on the surface streamline from opposite directions, they meet at a point and then depart from the wall. This phenomenon is called 'flow separation'. If the fluid moves towards the surface, the flow direction is reversed, and an 'attachment point' is obtained. A common name for separation and attachment point is 'stagnation point', since the fluid near these points is stagnant.

Steady separation of a two-dimensional boundary layer was explained first by Prandtl (1904), in which he introduced the boundary layer theory. Fluid particles near the surface are retarded by the friction of the wall and by an adverse pressure gradient present in the free stream. If the near-wall fluid has insufficient momentum for it to continue its motion, it will be brought to rest at the separation point. Further downstream, the adverse pressure forces will cause reverse flow. Since the velocity at the wall is always zero, the gradient must be positive upstream of the separation, zero at the point of separation, and negative in the reverse flow region.

Fluid particles in the boundary layer are subject to two important effects associated with the external pressure gradient and the moving wall. When the wall is stationary, fluid particles near the surface are slowed to rest to satisfy the no-slip condition at the surface; such particles are especially susceptible to the influence of the external pressure field, and in a zone of adverse pressure gradient, flow reversal, leading to zones of recirculation and ultimately boundary-layer separation, can easily occur. Once the wall moves, however, the fluid particles near the surface are energized and become less susceptible to the effects of the external pressure field. Separation can still occur under such circumstances, but it originates at an increasing distance from the wall with increasing wall speed where the particle speeds are generally smaller.

The separation for steady flow past fixed walls is defined as being associated with the development of reverse flow and the location where the wall shear stress vanishes. The solution of the steady, two-dimensional laminar boundary layer equation with a prescribed external pressure (or external velocity) distribution breaks down at the point of separation. This is commonly known as the Goldstein's singularity. For certain steady boundary layer flows past fixed walls , a self consistent asymptotic structure can be constructed using interactive boundary layer concepts (Smith, 1982).

Smith (1986) divided the steady two-dimensional laminar separations into two categories: The larger-scale break way separations, and the smaller-scale regular flow reversals. The former are more violent and characteristic of grossly distributed flows, while the latter are calmer phenomena associated with smaller disturbances. The large-scale or break way separation is a sudden localized process involving the increasing shift, away from the smooth body surface of the main part of the oncoming boundary layer.

Separation for unsteady flows and steady flows past moving walls are more complicated. Unsteady boundary-layer flow separation at high Reynolds numbers occur in an extensive number of flow configurations. For example, flow past surface-mounted obstacles, flow on the suction side of a thin airfoil at an angle of attack of the mainstream, vortex induced separation, vortex shedding behind bluff bodies and flow in internal passage with obstructions. Moore (1958), Rott (1956) and Sears (1956) concluded independently that vanishing wall shear and reversed flow were not a general criteria for unsteady separation. In Moore-Rott-Sears (MRS) model for steady flow over a moving wall, at separation, a Goldstein type singularity occurs at the location where the velocity profile has simultaneously zero velocity and shear at a point above the moving wall. Equivalently, for unsteady separation on a fixed wall, the separation point in the location at which both the shear and velocity vanish in a singular manner in a frame of reference moving with the separation point. Subsequently, Sears and Telionis (1975) defined separation in unsteady flow as that instant at which a singularity evolves in the solution of the boundary layer equations. In physical terms, this breakdown of the boundary layer solution indicates the first instant when a hitherto thin and passive shear layer adjacent to the wall starts to interact with the external flow and thereby separate from the surface. Sears and Telinois (1975) also proposed that the unsteady separation singularity would be similar to the Goldstein singularity.

Some controversy still exists over the precise definition of unsteady separation. Telionis (1979) reiterated that separation means the location on the solid boundary where the flow stops creeping over the skin of the body and breaks away from the wall, thus generating a turbulent wake. Unsteady boundary-layer flow separation at high Reynolds number in different flow configurations was studied by Van Dommelen (1981), Van Dommelen et al. (1990) and Doligalski et al. (1994). Gad-el-Hak and Bushnell (1991a) made a detailed review on unsteady separation. Recently, Degani et al. (1998) studied unsteady boundary-layer development over moving walls in the limit of Reynolds numbers using both the Eulerian and Lagrangian formulations. There, they gave an explanation on physical mechanism through which separation and a strong interaction with the external flow are suppressed.

1.2 Vortex Formation

Vorticity may be interpreted as the angular velocity of the fluid at a point in space. Angular velocity is a vector normal to the plane of rotation (thus it coincides with the axis of rotation).

Vorticity is produced at a body surface and spreads from there into the fluid. At the separation point (and at the re-attachment point) the vorticity is zero. Since the vorticity produced at the walls depends on the increase in velocity perpendicular to the wall, vorticity at the wall decreases if the streamlines near the wall diverge. At the separation point, vorticity of the boundary layer is carried into the fluid in the form of a shear layer. The occurrence of flow separation is, therefore, a prerequisite for the generation of vortices at solid walls.

Flow separation at the sharp edge of a body leads to a convection of vorticity into the fluid. Flow always separates at the sharp edge of a body, except at an edge parallel to the flow. This behavior is caused by the large curvature of the edge. Upstream of the edge, the streamlines converge so much that near the edge a large amount of vorticity is produced. This vorticity moves away through convection and forms a horseshoe vortex. However, flow separation does not necessarily require a convection process of vorticity.

Vortices moving close to a plane wall may induce a separation and give rise to a vortex known as secondary vortex, the vorticity of which is opposed to that of the primary vortex. The size of the secondary vortex increases quickly, thus pushing up the primary vortex; this is the mechanism of rebound. Several experimental, theoretical and numerical studies have been made on the vortex-induced separation because of its wide range of applications. Gad-el-Hak and Bushnell (1991b) and Rockwell (1998) made a detailed review on vortex induced separation and vortex body interactions.

1.3 Vortex Shedding

Since ancient times, it has been known that wind causes vortex induced vibration of the wires of an Aeolian harp. In the fifteenth century, Leonard da Vinci sketched a row of vortices in the wake of a piling in a stream. In 1878, Strouhal found that the Aeolian tones generated by a wire in the wind were proportional to the wind speed divided by the wire thickness.

As fluid flows towards the leading edge of a cylinder, the pressure in the fluid rises from the free stream pressure to the stagnation pressure. Near the widest section of the cylinder, the boundary layer separates from each side of the cylinder surface and form twin vortices behind the cylinder. Since the innermost portion of the shear layers, which is in contact with the cylinder, moves much more slowly than the outermost portion of the shear layers, which is in contact with the free flow, the shear layers roll into the near wake, where they fold on each other and coalesce into discrete vortices. A regular pattern of vortices called a vortex street, trials aft in the wake. New vortices form in an alternating way, and they also detach. In distinction to the steady flow, a periodic asymmetric flow is generated, which is called a 'Karman vortex street' after the fluid dynamicist Von Karman.

The concept of vortex separation can be distinguished from the flow separation in several ways. Vortex separation is always a time dependent process, in which vorticity assumes external values inside the fluid. By contrast, flow separation occurs in time dependent as well as in steady motion. Through inviscid analysis it can be shown that a symmetric vortex street is unstable. The ratio of vertical to horizontal distance of the vortices must be about 0.28 in the stable state at beginning of the vortex street, and this ratio becomes larger downstream during the decay of the vortices.

The physical mechanism for vortex formation in bluff body wake was studied by Gerrard (1966). Gerrard (1966) suggested that a forming vortex draws the shear layer (of opposite sign) from the other side of the wake across the wake center line, eventually cutting off the supply of voticity to the growing vortex. At the start of the motion, the wake cavity contains a symmetrical pair of equal and opposite recirculating fluid regions on either side of the wake. However when the vortices begin to shed, the cavity opens and forms an instantaneous alleyways through which fluid penetrates the cavity.

The frequency with which the vortices are shed from the body can be made dimensionless with the flow velocity and the characteristic length of the body. The non-dimensional parameter which measures the shedding frequency is known as Strouhal number (St), named in honor of the physicist Strouhal. Thus, the Strouhal number depends only on the Reynolds number.

1.4 Navier-Stokes Equations

The Navier-Stokes equations of motion for compressible fluid without any body force in vector notations are as follows

$$\frac{D\vec{q}}{Dt} = -\vec{\nabla}\int\frac{dp}{\rho} + \frac{4}{3}\nu\vec{\nabla}(\vec{\nabla}.\vec{q}) - \nu\vec{\nabla}\times(\vec{\nabla}\times\vec{q}), \tag{1.1}$$

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}.(\rho\vec{q}) = 0, \tag{1.2}$$

where

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{q}) = \vec{\nabla}(\vec{\nabla}.\vec{q}) - \vec{\nabla}^2\vec{q}.$$

For incompressible fluid ρ is constant and div$\overrightarrow{q} = 0$, i.e. $\overrightarrow{\nabla}.\overrightarrow{q} = 0$. So the governing equations for an incompressible viscous fluid are

$$\frac{D\overrightarrow{q}}{Dt} = -\frac{1}{\rho}\overrightarrow{\nabla}p + \nu\overrightarrow{\nabla}^2\overrightarrow{q}, \tag{1.3}$$

$$\overrightarrow{\nabla}.\overrightarrow{q} = 0. \tag{1.4}$$

In the above equations $\overrightarrow{q}, p, \rho, \nu$ are the vector velocity field, the pressure, the density and the kinematic viscosity, respectively. The total derivative notation as used above is given by

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \overrightarrow{q}.\overrightarrow{\nabla}$$

with

$$\overrightarrow{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

in Cartesian coordinates.
Thus, equation (1.3) is a vector equation expressing the change in momentum of a fluid element due to pressure and viscous force. Equation (1.4) is the continuity equation and expresses the conservation of mass for an incompressible fluid element.

The solution of the above equations becomes fully determined physically when the boundary and initial conditions are specified. In the case of viscous fluids the condition of no-slip on solid boundaries must be satisfied, i.e. on a wall both the normal and tangential components of the velocity must vanish.

1.5 Streamlines

The analytical description of flow velocities by the Eulerian approach is geometrically depicted through the concept of streamlines. In the Eulerian method, the velocity vector is defined as a function of time and space coordinates. If for a fixed instant of time, a space curve is drawn such that the direction of the tangent at every point of the surface coincides with the direction of velocity vector, then this curve is called a streamline. Therefore, the Eulerian method gives a series of instantaneous streamlines of the state of motion. In other wards, a streamline at any instant can be defined as an imaginary curve or line in the flow field so that the tangent to the curve at any point represents the directions of the instantaneous velocity at that point. In an unsteady flow where the velocity vector changes with time, the pattern of streamlines also changes from instant to instant. In a steady flow, the orientation or the pattern of streamlines will be fixed. From the above definition of streamline, it can be written

$$\vec{q} \times d\vec{s} = 0 \tag{1.5}$$

where $d\vec{s}$ is the tangent of an infinitesimal line segment along a streamline at a point, and $\vec{q}$ is the instantaneous velocity vector. The above expression, therefore, represents the differential equation of a streamline. In a cartesian coordinate system, the vectors $\vec{q}$ and $d\vec{s}$ can be written in terms of their components along the coordinate axes as $\vec{q} = \hat{i}u + \hat{j}v + \hat{k}w$ and $d\vec{s} = \hat{i}dx + \hat{j}dy + \hat{k}dz$. Then equation (1.5) gives

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \tag{1.6}$$

and thus describes the differential equation of streamlines in a cartesian frame of reference.

1.6 Vorticity Vector

We now consider flows for which curl $\vec{q} \neq \vec{0}$. The vector

$$\vec{\zeta} = \vec{\nabla} \times \vec{q} \tag{1.7}$$

is called the vorticity vector. Therefore, for an irrotational flow, vorticity components are also zero. A vortex line is a curve drawn in the fluid such that the tangent to it at every point is in the direction of the vorticity vector $\vec{\zeta}$. Therefore, the general equation of the vortex line can be written as,

$$\vec{\zeta} \times d\vec{s} = \vec{0}. \tag{1.8}$$

If the cartesian components of $\vec{\zeta}$ are $[\zeta_x, \zeta_y, \zeta_z]$, then the equations of vortex lines are given by

$$\frac{dx}{\zeta_x} = \frac{dy}{\zeta_y} = \frac{dz}{\zeta_z}. \tag{1.9}$$

1.7 Stream Function

The concept of stream function is a direct consequence of the principle of continuity. Let us consider a two-dimensional incompressible flow parallel to the xy plane in a rectangular cartesian coordinate system. The flow-field in this case is defined by

$$u = u(x,y,t), v = v(x,y,t), w = 0.$$

The equation of continuity is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1.10}$$

If a function $\psi(x,y,t)$ is defined in the manner

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{1.11}$$

so that it automatically satisfies the equation of continuity (1.10), then the function ψ is known as stream function. For a steady flow, ψ is a function of two variables x and y only. In case of a two-dimensional irrotational flow,

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 \tag{1.12}$$

so that

$$\frac{\partial}{\partial x}\left(-\frac{\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial y}\right) = 0$$

or,

$$\nabla^2 \psi = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 \tag{1.13}$$

Thus, for an irrotational flow, stream function satisfies the Laplace's equation.

Since ψ is a point function, it has a value at every point in the flow field. Hence, a change in the stream function ψ can be written as

$$d\psi = \frac{\partial \psi}{\partial x}dx + \frac{\partial \psi}{\partial y}dy = -vdx + udy$$

Further, the equation of a streamline is given by

$$\frac{u}{dx} = \frac{v}{dy} \quad \text{or} \quad udy - vdx = 0$$

It follows that $d\psi = 0$ on a streamline, i.e., the value of ψ is constant along a streamline. Therefore, the equation of a streamline can be expressed in terms of stream function as

$$\psi(x,y) = \text{constant} \tag{1.14}$$

Once the function ψ is known, streamline can be drawn by joining the same values of ψ in the flow field.