Abstract

In the present thesis we investigate the problem of estimating location and scale parameters in location scale families of distributions from a decision theoretic point of view.

In **Chapter 1**, we give a brief introduction to the problem of estimating location and scale parameters and a detailed review of the existing literature on this estimation problem.

In **Chapter 2**, some basic definitions, results and important approaches for improving equivariant estimators are discussed.

In **Chapter 3**, estimation of the scale parameter in a scale mixture of location–scale families under a quadratic loss function is considered. The technique of Strawderman (1974) has been extended to obtain a class of estimators improving upon the best affine equivariant estimator of the scale parameter under certain conditions. Further, integral expression of risk difference (IERD) approach of Kubokawa (1994) is used to derive similar improvements for the reciprocal of the scale parameter. Using the improved estimators of the scale parameter and the reciprocal of the scale parameter, classes of improved estimators of the ratio of scale parameters of two populations have been derived. In particular, Stein–type and Brewster–Zidek type estimators are derived for the ratio of scale parameters of two mixture models. These results are applied to the scale mixture of exponential distributions. In particular, this includes multivariate Lomax and modified Lomax distributions.

In **Chapter 4**, the estimation of the mean vector of a scale mixture of multivariate normal distributions under a balanced loss function, when the covariance matrix is identity, is considered. A class of estimators is proposed and it is shown that these estimators are minimax for this problem. This minimaxity result is further used to obtain a class of Bayes minimax estimators for the mean vector.

In **Chapter 5**, we consider the estimation of the common hazard rate parameter of two or more exponential distributions with unknown location parameters under a general class of bowl-shaped scale invariant loss functions. We have considered two types of problems. First we have considered the estimation of common hazard rate of two exponential distribution with ordered location parameters based on various sampling schemes such as (i) i.i.d. sampling, (ii) record values, (iii) Type-II censoring (iv) Progressive Type-II censoring. We have shown that the best affine equivariant estimator is inadmissible by deriving an improved estimator. In addition a smooth improved estimator is obtained. In particular, the improved estimators are obtained for three special loss functions. Secondly we have studied the estimation of common hazard rate of several exponential distributions. Similar results are obtained in this case. Finally, numerical comparison of risk performance of the proposed improved estimators is carried out for both the problems.

In **Chapter 6**, consider a bivariate normal population with unknown means θ_1, θ_2, known variances and a known correlation coefficient ρ, where $\theta_1 \leq \theta_2$. The problem of estimation of (θ_1, θ_2) is studied when the loss function is taken to be the sum of squared errors. We have considered two cases, equal variances and unequal variances. In both cases classes of minimax estimators are derived. These estimators improve upon the usual estimators. A class of admissible estimators is obtained within this class. The minimaxity and admissibility of a generalized Bayes estimator is established. Finally, the risk performance of all proposed estimators is compared numerically.

In **Chapter 7**, improved estimation of the entropy in shifted exponential distributions under a general location invariant loss function is considered. At first the estimation of entropy of single population is considered. Inadmissibility of the best affine equivariant estimator (BAEE)

of entropy is proved by deriving a Stein (1964) type estimator. This estimator is simple but not smooth. We derive a smooth estimator which improves upon the BAEE of entropy. The IERD approach is also used to derive a class of improved estimators. Secondly we have studied the estimation of entropy of two exponential populations with a common scale parameter and ordered location parameters based on several sampling schemes namely (i) i.i.d. sampling, (ii) record values, (iii) Type-II censoring (iv) Progressive Type-II censoring. The BAEE is shown to be inadmissible by deriving two improving estimators. In particular, the improved estimators are also derived with respect to the squared error loss and the linex loss functions. For both the cases the risk performance of the improved estimators is compared numerically using simulations.

In **Chapter 8**, the problem of estimating parameters of a Pareto distribution is investigated under a general scale invariant loss function when the scale parameter is restricted to the interval $(0, 1]$. Techniques for improving equivariant estimators developed by Stein, Brewster-Zidek and Kubokawa are adopted to derive improved estimators. In particular improved classes of estimators are obtained for the entropy loss and a symmetric loss. Risk functions of various estimators are compared numerically using simulations.

Keywords: Best affine equivatiant estimator; Best location equivariant estimator; Best scale equivariant estimator; Location invariant loss function; Scale invariant loss function; Brewster–Zidek technique; Stein-Type estimator; Brewster–Zidek-type estimator; Integral Expression of Risk difference; Mixed estimator; Ordered parameters; Hazard rate; Entropy.