Introduction

Considerable literature on the propagation characteristics of electromagnetic field through hollow pipes having cross-section in the form of regular geometrical figures like rectangle, circle and ellipse [1]-[4] as well as other shape such as single ridge rectangular waveguide, Pacman, Meinke waveguide [5] etc. is available. Guiding structures consisting of two conductors like strip line, suspended strip line, coaxial line and twin wire line are widely used principally for propagation of TEM/quasi TEM modes [6]-[9]. Twin wire line has been designed and used widely for connecting Television antenna to the input of the receiver. Coaxial lines are used for transmission of signals upto microwave frequency band.

Introduction of a lateral offset in the centre conductor of a coaxial line provides a simple way to decrease its characteristic impedance without modifying the dimension of the conductors [10]-[12]. This technique can be used to realise quarter-wave matching element which forms one of the sections in multisection quarter-wave transformer for broad-band matching [13].

In view of this important and interesting applications, the investigations of this type of coaxial line with displaced inner cylinder initiated interest of research workers for a long time [10, 12],[14]-[22]. In all these investigations, attention was paid to evaluate the higher-order mode cutoff frequencies of this line whose dominant mode is the TEM mode.

The evaluation of capacitance per unit length of a transmission line having (i) coaxial, (ii) eccentric annular and (iii) twin-wire configurations is available in the literature [2]-[4],[23, 24]. It would be of interest to find the lower limit of the length of the line for which the formulas derived using infinite length approximation for TEM mode analysis is valid. It has been shown by Harrington [25] using moment method formulation that the fringing at the ends of a pair of parallel plates modifies the capacitance as the size of the plates becomes smaller and smaller. He has not suggested any method for finding the effect of fringing in the cases of finite length of (i) coaxial line (ii) eccentric coaxial line and (iii) twin wire cylinders. In the case of two conductor transmission line consisting of conductors of infinite length, the TEM mode analysis is based on equal and opposite charges on the two conductors irrespective of whether their cross-sectional areas are equal or not. On the other hand, in the case of pair of conductors of finite length, the charges on the conductors are not equal in those cases where the surface areas of the conductors are different [26](page 194-195). Evaluation of capacitance between the conductors in the above configurations of cylinders of finite length taking into account this difference in total charge requires a completely different method of analysis. It is worthwhile to develop the method of analysis for finding a solution to this problem. The analysis presented by Harrington [25] for parallel plate using the method of moments can be extended to the case of single isolated square plate of finite size in the limit distance of separation between the plates tends to infinity. Similar technique can be applied for finding the capacitance of a single isolated cylinder of finite axial length from the formulation of a pair of cylinders of finite length in the limit the distance of separation between the cylinders tends to infinity.

Smythe [23] and Grivet [24] presented the TEM mode analysis of transmission line consisting of parallel cylinders for the following cases:
(i) one cylinder is inside the other and they are not concentric and (ii) one cylinder is outside the other and their radii are different. The case (i) represents an eccentric annular guide and the case (ii) includes the conventional two wire line of parallel cylinders with unequal radii. Smythe [23] and Grivet [24] used logarithmic transformation and expressed the variables in the transformed plane in terms of potential and flux functions. In the analysis carried out by them, they derived a set of two equations from which it was possible to derive only an expression for the difference in potential in terms of geometrical parameters of the line. From the knowledge of difference in potential, the expression for the capacitance per unit length of eccentric annular guide was obtained by them.

In an eccentric annular guide, the inner cylinder has to be supported on a dielectric block for the purpose of fabrication. It is essential to carry out the analysis for evaluation of the modified electrical parameters of the line with dielectric support for propagation of TEM mode. This dielectric support may fill the space between the two cylinders either partially or completely. Evaluation of the modification of capacitance in the case of eccentric annular guide which is essential from practical point of view is quite complex. It is worthwhile to point out that Naiheng *et al.* [27] carried out analysis for taking into account the effect of partial dielectric filling only for the case of coaxial line. It is therefore essential to find the effect of partial dielectric filling in the case of eccentric annular guide. The partially filled dielectric between the inner and outer cylinders may be provided in two ways (i) outer boundary of the dielectric is concentric with the inner cylinder and it touches the outer cylinder at a point (ii) outer boundary of the dielectric follows the boundary of the outer cylinder. The method of analysis presented by Smythe [23] and Grivet [24] for the derivation of capacitance per unit length of airfilled eccentric annular guide is not convenient for finding the change in capacitance in the presence of dielectric support between the inner and outer cylinders. This is because the formulation presented by them does not permit determination of transformation of the equipotential boundaries as an explicit function of geometrical parameters of the line. It is worthwhile to point out that the determination of this functional relation is useful for considering the effect of dielectric support, evaluation of field intensity in the region between these conductors, charge distribution on the conductors and estimation of cutoff frequencies of higher-order modes. Hence, it is essential to carry out investigation to obtain a transformation and its characteristic features which gives mathematical function expressing the transformed contour as an explicit function of geometrical parameters of the transmission line.

Smythe and Grivet also found an expression for the capacitance per unit length of a twin wire line of unequal radii (equation (5), page 78 of ref.[23] and equation (17), page 96 of ref[24]). When the general formula for the capacitance of two cylinders of unequal radii is used to derive the capacitance between the two cylinders of equal radii, the resulting expression for the capacitance obtained from their formula is found different from the one available in the literature [28, 29]. Making considerable simplifying approximations Smythe [23] derived the expression for the capacitance of parallel cylinders of equal radii which is the same as that available in the literature [28, 29]. Hence, it is worthwhile to present a formulation which is free from this limitation and enables evaluation of capacitance of parallel cylinders of equal radii from general formulation for pair of cylinders of unequal radii without making any approximation. In this case also, using the formulation of Smythe and Grivet, the determination of transformation of equipotential boundaries as an explicit function of radii and distance between the centres of the cylinders is quite involved.

For the purpose of providing insulation, the conductors used in the transmission line of parallel cylinders are covered with dielectric. The limitation of the formulation of Smythe [23] and Grivet [24] for taking the effect of dielectric support has already been discussed. Clements *et al.* [29] presented the formulation for finding the capacitance matrix of a system of dielectric coated wires using the method of moments. For a pair of dielectric coated wires of equal radii they presented numerical results on capacitance per unit length for the particular case the width of the dielectric is equal to the radius of the cylinders and the dielectric coatings touch each other. This touching dielectric case is of significance importance. In the formulation [29], Clements *et al.* expressed the total circumferential charge (bound and free) on the conductor and dielectric surfaces in terms of suitable expansion functions and applied the boundary conditions on potentials at points on the conductor surfaces and boundary conditions on the normal displacement vector at the dielectric surface leading to a matrix equation for the unknown total charge densities. They have studied three different techniques including (i) Polygonal representation of the boundary and pulse expansion functions (ii) exact boundaries and pulse expansion function (iii) exact boundaries and harmonic expansion function. The authors of ref. [29] stated that the third method is advantageous compared to the first two techniques. In this technique, the set of unknown charge coefficients were evaluated by truncating the harmonic series and from the solution of the matrix equation. In view of the obvious complexity involved, they presented numerical data on capacitance for the particular case, the width of the dielectric is equal to the radius of the individual cylinders. They did not present any data on the variation of capacitance with the width of the dielectric. Application of their method for more general case of unequal cylinders with arbitrary width of dielectric involves considerable complexity. Further when the pair of conductors is covered by a dielectric strip for mechanical rigidity, as in television cable, application of the technique developed by Clements *et al.* [29] is found to be more complex. It is worthwhile to explore the possibility of developing a formulation which leads to more or less a closed form expression for the capacitance in the presence of dielectric coating on the individual conductor and dielectric strip above both the conductors for the purpose of mechanical rigidity.

Any change in capacitance modifies the characteristic impedance and also the velocity of propagation [27, 30]. When a pair of dielectric coated cylinders are placed over a perfectly conducting ground plane, there is a further modification of capacitance because of the presence of ground plane. This aspect was not investigated by Clements *et al.* [29]. For a pair of dielectric coated cylinders above a ground plane, it is useful to see the variation of capacitance, characteristic impedance and velocity of propagation with varying 'D', the distance of separation between the axes of the cylinders. The configuration for which it is essential to find all these electrical parameters is shown in Figure 0.1 for the sake of clarity.

It is therefore essential to develop a formulation for finding the variation of electrical parameters for the structure of Fig. 0.1. The concept and formulation developed can then be extended to find the effect of dielectric substrate on the electrical parameters. The corresponding configuration is shown in figure 0.2

The range of variation of the dielectric constant of the substrate material for which the electrical parameters have to be evaluated are $2, 4, 6, \ldots, 12.9$. Considering the importance of this result in the high frequency range and also in the low frequency application when two different size wires are placed on a dielectric substrate, it is useful to develop a formulation for finding the electrical parameter for the configuration of Figure 0.2.

When operating relatively at high frequencies, higher order modes are excited in