CHAPTER - I

INTRODUCTION

In this thesis we have developed the constitutive equations to a class of materials which we term as 'Thermo-hypoelastic Materials' from which classical thermoelastic materials are shown to be the first order approximation.

Hypoelastic materials, without temperature considerations were first considered by Truesdell [1955] and later further developed by Noll [1955], Ericksen [1958], Bernstein [1958], [1960] and Green [1956], [1967]. A brief account of these developments is given in section 1.3. However, before embarking on it, we briefly outline the various concepts and principles of continuum mechanics which have to be satisfied for all unified continuum theories. Needless to add, the constitutive theory of Thermo-hypoelastic materials which we develop in Chapter II, satisfies all these. We briefly recount these in the following.

1. BASIC CONCEPTS OF CONTINUOUS MEDIA

The theory of continuous media is a scientific discipline concerned with the global behaviour of substances under the influence of external 'disturbances'. External agents that produce changes in the state of the medium may be surface or body forces or may be contact forces such as heat, and electrical, chemical, mechanical or any other type of disturbances. The nature of these forces, the laws governing them, their mathematical characterization and their physical measurements must be an integral part of any program of study in the theory of continuous media.

1.1 CONSTITUTION OF THE MEDIUM

It is generally observed that the response of different substances of the same geometry when subjected to the same external forces is different. This is caused mainly because of the differences in the constitution of the various substances. Therefore the constitution of bodies plays an essential role in the description of physical phenomena. From a continuum view point constitution must be understood in a macroscopic sense. The theory of continuous media must define the medium in accordance with the scope it intends to encompass. At the beginning of this century many mathematicians had attempted for a unified theory of continuous media. The works of Jauman [1911] and Lohr [1917] illustrate the point towards this end.

The most important fact in this approach is that the medium should be continuous one. This means that the field quantities (e.g. displacements, stresses, electric fields etc.) are piecewise continuous functions of the coordinates of the material point and the time.

The ultimate interest in physical problems of Continuum Mechanics is to predict the way in which the medium responds to the external forces. The unified approach to the study of the global behaviour of materials consists of first a thorough study of basic principles common to all media and second, a clear demonstration of the types of media such as elastic solids, stokesian fluids, hygrosteric materials within the structure of the theory. The theory so constructed makes available methods which are useful in the creation of new fields of research. Topics like Hypoelasticity, non-Newtonian fluids, Electroelasticity, Thermo-viscous fluids illustrate that the description of a physical phenomena is introduced through the constitutive equations given by functional equations. These considerations indicate that in the design of a theory of continuous media there exist two definite expositions constituting the backbone of the theory: (1) basic principles and (2) a constitutive theory.

THE BASIC PRINCIPLES

These are the fundamental axioms essential in the construction of the foundations of the theory. The domain of applicability of these principles determines the strength of the theory. These principles are not certainly unchangeable but any change in these principles must arise from the failure of the theory. The basic principles upon which the theory is constructed are:

(a) CONSERVATION OF MASS

$$\frac{\partial \rho}{\partial t} + (\rho\ v^k)_{,k} = 0 \qquad (1.1.1)$$

where ρ is the mass density, t is the time and $v^k = \dot{x}^k$ the velocity vector. The comma denotes partial differentiation with respect to coordinates in the deformed body and dot denotes partial differentiation with respect to time t.

(b) BALANCE OF LINEAR MOMENTUM

$$S^{jk}_{,j} + \rho\ (f^k - \dot{v}^k) = 0 \qquad (1.1.2)$$

where S^{jk}, f^k, $\dot{v}^k$ are respectively the components of stress tensor S, body force f and acceleration $\dot{v}$.

(c) BALANCE OF MOMENT OF MOMENTUM

$$S^{jk} = S^{kj} \qquad (1.1.3)$$

(d) CONSERVATION OF ENERGY

$$\rho\ r - \rho\ \dot{U} + \mathrm{tr}\ (s\ d) - \mathrm{div}\ q = 0 \qquad (1.1.4)$$

where $\dot{U}$ is the time rate of internal energy U, q is the heat flux vector, r the source of energy, S the stress tensor and d the rate of deformation tensor.

(e) THE PRINCIPLE OF ENTROPY

$$\frac{d}{dt} \int_B \rho \eta \; dV + \oint_S \frac{\bar{h} \cdot \bar{n}}{\theta} \, dA \geq 0 \qquad (1.1.5)$$

where η the entropy density function, $\bar{h}$ the heat flux bivector, $\bar{n}$ the unit normal and θ the local temperature.

CONSTITUTIVE THEORY

The basic principles explained above are valid for all materials irrespective of their constitution. It is expected that their mathematical expressions are not sufficient to predict uniquely the behaviour of all substances under prescribed boundary and initial conditions. In order to take account of the nature of different materials, we must therefore find additional equations identifying the basic characteristics of the body with respect to the response sought. There are many approaches to this problem namely, (a) The Statistical Approach, (b) Purely Mathematical Approach