Chapter 1

Introduction

1.1 Non-Newtonian Fluid

Any fluid that does not obey the linear relationship between the shear stress τ and shear rate $\dot{\Upsilon}$ is termed as non-Newtonian fluid. Liquids with high molecular weight which include polymer melts and solutions of polymers, as well as liquids in which fine particles are suspended (slurries and pastes), are non-Newtonian. In this case slop of the shear stress versus shear rate curve will not be constant, it changes with shear rate. Shenoy and Mashelkar (1982) classified non-Newtonian fluids into two broad categories: (I) inelastic fluids and (II) viscoelastic fluids, and subdivided inelastic fluid into (a) time-independent and (b) time-dependent fluids.

Inelastic Non-Newtonian Fluid

(a) Time Independent non-Newtonian Fluid

For time independent non-Newtonian fluid, there is a non-linear relation between the stress and rate of strain which is written as:

$$\tau = f\left(\dot{\Upsilon}\right) \tag{1.1}$$

Depending upon the form of the function in equation (1.1), these fluids are further subdivided into three types:

(i) pseudoplastic

(ii) viscoplastic

(iii) dilatant

Pseudoplastic fluids often referred to as shear-thinning fluids, exhibit a viscosity decrease with increasing shear rate. Some of the pseudoplastic fluids are Polymer solutions, polymer melts, printing inks and blood. Dilatants often referred as shear-thickening fluids, exhibit viscosity increase with increasing shear rate. Some of dilatants fluids are gum solution, aqueous suspensions of titanium dioxide, wet sand, starch suspensions. Shear-thinning behavior is more common than shear-thickening. The power law fluid, Bingham fluid, Herschel-Bulkley fluid, Casson fluids are the examples of this class.

The Ostwald-de Waele Power law model

The constitutive equation for the power-law fluid (Ostwald (1925),Waele (1929)) is as follows:

$$\begin{aligned} \tau &= \mu^* \dot{\Upsilon}^n \\ &= \mu^* |\dot{\Upsilon}|^{n-1} \dot{\Upsilon} \end{aligned} \tag{1.2}$$

It has effective viscosity or apparent viscosity $\mu^* \dot{\Upsilon}^{n-1}$ and μ^* reflects the consistency of the fluid and n is the power-law index. When $n < 1$, the model describes pseudoplastic behavior, while $n > 1$, dilatant behavior and for $n = 1$, Newtonian behavior.

Bingham Plastics

The constitutive equation for Bingham fluid (Bingham (1922)) is as follows:

$$\begin{aligned} &\tau = \mu_p \dot{\Upsilon} + \tau_y, \quad && \tau \geq \tau_y \\ &\dot{\Upsilon} = 0, && \tau < \tau_y \end{aligned} \tag{1.3}$$

It exhibits a yield stress τ_y at zero shear stress, followed by a linear relationship between shear stress and shear strain rate, the plastic viscosity is μ_p, being the slope of the straight line. If $\tau < \tau_y$, no flow takes place. Water suspension of clay, fly ash, metallic oxides, jellies, tomato ketchups, toothpastes and paints are Bingham plastics.

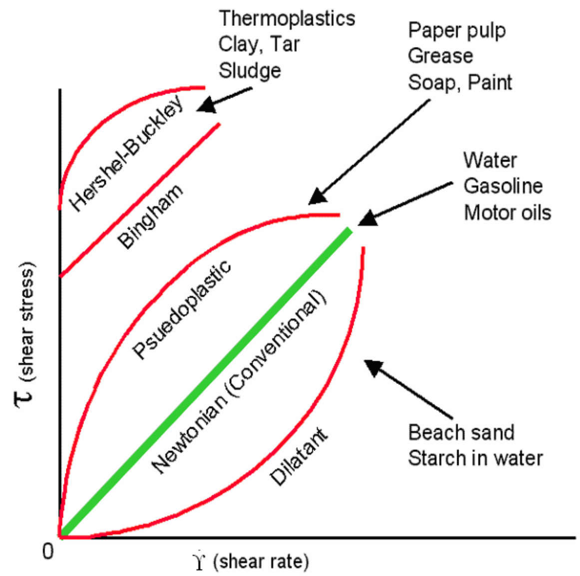

Fig. 1.1: Rheological model of non-Newtonian fluid (http://www.technet.pnl.gov)

Herschel-Bulkley fluid

The Herschel-Bulkley fluid (Herschel and Bulkley (1926)) is a generalized model of a non-Newtonian fluid, in which the shear stress experienced by the fluid is related to the strain in a complicated, non-linear way. The relation of the shear stress and shear rate is written as:

$$\begin{aligned} &\tau = \mu * \dot{\Upsilon}^{n} + \tau_y, && \tau \geq \tau_y \\ &\dot{\Upsilon} = 0, && \tau < \tau_y \end{aligned} \qquad (1.4)$$

The flow behavior index n and yield stress τ_y are the characteristics parameters. The flow index measures the degree by which the fluid is shear-thinning or shear-thickening. The velocity of the fluid at the core region is constant as $\dot{\Upsilon} = 0$ at that region. It can be noted that when yield stress is absent this model reduces to Ostwald-de Waele power law model, and when $n = 1$ it reduces to Bingham plastic model.

(b) Time dependent Fluids

Some fluids are more complex because the apparent viscosity depends not only on the strain rate but also on the time the shear has been applied. These can be generally classified into two classes as (i) thixotropic fluids and (ii) rheopectic fluids. Thixotropic fluids shows a decrease in shear stress with time as the fluid sheared while rheopectic fluids exhibits a increase in shear stress with the time as the fluid sheared.

Viscoelastic Fluids

A viscoelastic material exhibits both elastic and viscous properties. The simplest viscoelastic fluid is one which is Newtonian in viscosity and obeys Hooke's law for the elastic part by given constitutive equation

$$\dot{\Upsilon} = \frac{\tau}{\mu} + \frac{\dot{\tau}}{\kappa} \tag{1.5}$$

where κ is a rigidity modulus. When a viscoelastic fluid flows, certain energy is stored up in the material as strain energy in term of viscous dissipation. Walters liquid B, Oldroyd B fluid, Rivlin-Ericksen fluid etc., are the examples of some viscoelastic fluids.

1.2 Porous Medium – Heat and Mass Transfer

A porous medium may be defined as a solid matrix containing holes either connected or non-connected, dispersed within the medium in regular or random manner provided such holes occur frequently in the medium. If these pores are saturated with a fluid, then the solid matrix with the fluid is called a fluid saturated porous medium. The saturating fluid may be Newtonian or non-Newtonian. The flow of the fluid in a saturated porous material is possible only when some of the pores are interconnected. The interconnected pore space is termed as the effective pore space and the whole of the pore space is called the total pore space. If the package of the solid grains is regular, then it is called an ordered porous material, otherwise it is called a random porous material. Most natural and some of the artificial porous materials have random void structure. Natural porous media are ground soil, beach sand, rye bread, wood, human lung, etc., a few to quote. Porosity of a porous material is defined as the fraction of the bulk volume of the porous material occupied by pores. This gives the total porosity of the medium. But the effective porosity φ is defined as fraction of the bulk volume of the material occupied by the interconnected pores. Depending on the structure of the porous medium, the fluid conductivity of the porous material i.e., the permeability K is defined as the ease with which a fluid may be made to pass through the material by an applied pressure gradient. The permeability depends on the microstructure of the medium and is independent of the properties of the saturating fluid. Connecting permeability with the fluid property μ as $\frac{K}{\mu}$ gives the mobility of the fluid in the medium.

Heat propagation means the exchange in internal energy between individual elements or regions of the medium considered. It always occurs from the higher temperature region to the lower temperature region. There are three modes by which heat transfer is possible. Those are Conduction, Convection and Radiation. Conduction is because of molecular transport of heat in bodies (or between bodies) in the thermo dynamical system considered. There is no actual displacement of particles from one place to another. Convection is concerned with the fluid medium and/or the fluid in the medium. The motion of a non-isothermal fluid is called convection. Here, the transport of heat is mainly because of the movement of fluid from one region to the other region in the medium. Pure conduction can be observed in solids where as heat transfer by convection is always accompanied by conduction and this is observed in fluid media. The conversion of the internal energy of a substance into radiation energy is referred to as radiation heat transfer. It propagates by means of electromagnetic waves depending on the temperature and on the optical properties of the emitter.

By mass transfer we mean the tendency of a component in a mixture to travel from a region of high concentration to one of low concentration. For example, if an open test tube with some water in the bottom is placed in room in which the air is relatively dry, water vapor will diffuse out through the column of air in the test tube. There is transfer of water from a place where concentration is high (just above the liquid surface) to a place where its concentration is low (at the outlet of the tube). If the gas mixture in the tube is stagnant, the transfer occurs by molecular diffusion. If there is a bulk mixing of the layers of gas in the tube by mechanical stirring or because of a density gradient, mass transfer occurs primarily by the mechanism of forced or natural convection. These mechanisms are analogous to the transfer of heat by conduction and by convection; there is, however, no counterpart in mass transfer for thermal radiation.

Convective heat transfer phenomena in nature are often accompanied by mass transfer, that is, by the transport of substances that act as components (constituents, species). Convective heat and mass transport is further classified as Forced Convection, Free Convection and Mixed Convection. Forced Convection is due to an external agent unrelated to heating effect, which induces the flow of fluid over the heated body where as the motion in natural convection arises only because of density variations which come into play due to temperature and concentration changes in material phases and other effects in the body force field. The natural flow developed is relatively weak with relatively small velocities when compared with the forced flows. Also the governing equations will become coupled in the natural convection process where as in the forced convection process, flow field can be solved independently first and then used to solve the energy and concentration equations for finding the temperature and concentration distributions in the media. In mixed convection the order magnitude of the buoyancy force is comparable with the externally maintained pressure drop to force the flow. If the buoyancy force has component in the direction of the free stream, then it is called an aiding flow and if buoyancy opposes the free stream, then it is called an opposing flow.

Thus to understand the convective heat and mass transfer in a porous medium, the flow field in the porous medium has to be understood properly first. Attempts were made to understand the flow through such a complicated labyrinth in two ways, by postulation and averaging approaches. In the postulation approach one develops balance equations for each phase by writing the conservation laws directly in terms of the average quantities. Constitutive behavior, including the transport coefficients and the interactions between them, are then deduced from experiments. The final product is a closed set of equations in which the dependent variables are averages of the microscopic field variables such as velocity, pressure, temperature and concentration. In the averaging approach one starts by writing the microscopic balance equations for each phase in differential form. These are the familiar equations of fluid mechanics, heat and mass transfer. One then takes the phase average of each equation to produce an averaged balance equation. Most of the analytical studies in porous medium used the former approach.

1.2.1 Equation of Continuity

For the solid part, equation of continuity holds since the solid in the medium is stationary (we consider consolidated solid structure). For the fluid medium we can derive the equation of continuity as: the mass instantaneously trapped inside the control volume is equal to the net flow rate (in–out). This implies

$$\frac{\partial \rho}{\partial t} + \nabla . \rho \boldsymbol{V} = 0 \tag{1.6}$$

where ρ is the density of the fluid, and V is the volume averaged velocity vector. Usage of the volume averaged quantities in deriving this equation makes it to look similar to the equation of continuity for clear fluids. The concept of volume averaged velocity was introduced precisely in order to apply the pure fluid mathematical tools to the flows through porous media see Bejan (2004). When the density of the fluid is constant the above equation becomes

$$\nabla . \boldsymbol{V} = 0 \tag{1.7}$$

1.2.2 Equation of Motion

Modified Darcy Law for Power-Law Fluids

Detailed discussion on the derivation of Darcy law for Newtonian fluid can be found in Bejan (2004). Here we give the description of the modified Darcy law for the power law fluids. Christopher and Middleman (1965) were the first to propose the form for Darcy law applicable to power-law fluids. This was followed by Kemblowski and Michiniewich (1979), who reviewed the entire literature on fluid flow through granular beds and derived a new expression for Darcy law for power law fluids. Dharmadhikari and Kale(1985) performed extensive experiments and presented correlations. In essence, the modified Darcy law as obtained by Christopher and Middleman (1965), Kemblowski and Michiniewich (1979) and Dharmadhikari and Kale (1985) can be as:

$$\nabla p = \rho \boldsymbol{g} - \left(\frac{\mu^* |\boldsymbol{V}|^{n-1}}{K^*} \right) \boldsymbol{V} \tag{1.8}$$

where μ^* reflects the consistency of the power-law fluid and K^* is the modified permeability, defined as

$$K^* = \frac{1}{2c_t} \left(\frac{n\varphi}{3n+1} \right)^n \left(\frac{50K}{3\varphi} \right)^{\frac{n+1}{2}} \tag{1.9}$$

where n is the power law index of the fluid, φ is the porosity, K the intrinsic permeability:

$$K = \frac{\varphi^3 d^2}{150(1-\varphi)^2}. \tag{1.10}$$

In the above equation c_t is the tortuosity factor defined as:

$$c_t = \begin{cases} \dfrac{25}{12} & \text{Christopher and Middleman (1965)} \\ (2.5)^n 2^{(1-n)/2} & \text{Kemblowski and Michniewicz(1979)} \\ \dfrac{2}{3}\left(\dfrac{8n}{9n+3} \right)^n \left(\dfrac{10n-3}{6n+1} \right) \left(\dfrac{75}{16} \right)^{3(10n-3)/(10n+11)} & \text{Dharmadhikari and Kale (1985)} \end{cases}.$$

If the expression for c_t given by Dharmadhikari and Kale (1985) is used, the power law index throughout the text must be changed to a new power law index $n' = n + 0.3(1-n)$.

Based on the length scale of $(K/\varphi)^{1/2}$ taken from equation (1.10) and the average velocity in the interstices of a porous medium $u_E = (u/\varphi),$ the Reynolds number is defined for the flow of a power law fluid through porous medium:

$$\mathrm{Re}_K = \frac{\rho (K/\varphi)^{n/2} (u/\varphi)^{2-n}}{\mu^*}. \tag{1.11}$$

In deriving and using the equation (1.8), there is an implicit assumption that the flow is slow enough or the pores small enough to maintain a value of the Reynolds number Re_K much less than 1.

Darcy-Forchheimer Equation for power law Fluids

The validity of the modified Darcy law ceases when the Reynolds number exceeds 1. This occurs when flow enters a nonlinear laminar regime at which point porous inertia effects have to be considered. For a Newtonian fluid flow, Forchheimer (1901) proposed a square velocity term in addition to the Darcy velocity term to account for this effect, which Muskat (1949) called the Forchheimer term. This pioneering work was followed by other proposals for mathematically describing non-Darcy flow, such as those of Ergun (1952) and Ward (1969).

For non-Newtonian inelastic power law fluids, Shenoy (1993) derived the Forchheimer modification of the Darcy law. Instead of using the capillary model as had been done by Chirstopher and Middleman (1965) and Dharmadhikari and Kale (1985) for the pure Darcy case, Shenoy (1993) preferred to treat the porous medium as a body composed of individual discrete particles and completely filled, i.e., saturated with a non-Newtonian power law fluid. The expression derived by Shenoy (1993) can be written as follows:

$$\nabla p = \rho \boldsymbol{g} - \left(\frac{\mu^* |\boldsymbol{V}|^{n-1}}{K^*} + \frac{c\rho |\boldsymbol{V}|}{K^{1/2}} \right) \boldsymbol{V} \tag{1.12}$$

From (1.12) it can be noted that the Forchheimer term is the same for Newtonian as well as non-Newtonian fluids, and understandable, as it is basically the porous inertia term that is independent of viscous property effects. In equation (1.12), c is the inertia coefficient, as it reflects porous inertia effects (i.e. separation and wake effects) when they become relevant at higher flow velocities. When $c = 0$, i.e., when Forchheimer effects can be neglected, equation (1.12) reduces to the conventional Darcy law. However, when the flow is in the non linear regime, c assumes nonzero values as given by the expression

$$c = 0.143\varphi^{-1/2} \tag{1.13}$$

It is generally accepted that c is a function of the microstructure of the porous medium (Ergun (1952), Beavers and Spparow (1969)). Through extensive experiments in a large variety of porous media Ward (1969) found that c is a constant and approximately equal to 0.55 for Newtonian fluids.

1.2.3 Energy and Concentration Equations

Two options exist for deriving the energy equation in porous media. One is the two-phase model and the other, which is widely used, is local equilibrium model. Using local thermal equilibrium model the steady state energy equation is given by

$$\boldsymbol{V}.\nabla T = \nabla.(\alpha \nabla T) + \frac{\mu^*}{\rho K^* c_p}\boldsymbol{V}(|\boldsymbol{V}|^n + \frac{\rho_\infty b K^*}{\mu^*}\boldsymbol{V}^2) \tag{1.14}$$

where T is the local equilibrium temperature, $b(= c/\sqrt{K}$ in equation $(1.12))$ is the empirical constant associated with the Forchheimer porous inertia term, c_p is the specific heat at constant pressure and α is the thermal diffusivity of the medium. The second term in the equation (1.14) is the dissipation term for the power law fluid which is equal to the mechanical power needed to extrude the viscous fluid through the pore (Bejan (2004)), this power requirement is equal to the mass flow rate times the externally maintained pressure drop divided by fluid density.

The term mass transfer means the transport of concentration of a substance that is involved as a component in the fluid mixture, for example, the transport of salt in water or a chemical contaminant in fluid. Convective mass transfer is analogous to convective heat transfer. With C being the concentration of the solute, the convective steady state mass transfer in the porous medium is given by the equation

$$\boldsymbol{V}.\nabla C = \nabla.(D\nabla C) \tag{1.15}$$

where D is the solutal diffusivity. It can be seen that the steady state energy and concentration equations for flow through porous media is the same as that normally used in analyzing heat transfer in clear fluids.

In combined convective-radiative flows, the radiative heat flux can be approximated using Rosseland approximation depending on the size, shape of the solid emitter and the absorbing emitting properties of the porous medium. The Rosseland approximation for radiation is given by $q_y^r = -\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial y}$ (Sparrow and Cess (1978)), where σ, k^* are the Stefan-Boltzmann constant and the mean absorption coefficient respectively. When the temperature gradients and concentration gradients are very large, the mass flux due to temperature gradient (Soret effect) and heat flux due to concentration gradient (Dufour effect) might become significant. These effects are also known as thermo-diffusion and diffusion-thermo effects, which give coupling between temperature and concentration fields. With Soret and Dufour effects the modified energy and concentration equations in steady state can be written as

$$\boldsymbol{V}.\nabla T = \nabla.\left(\alpha_e \nabla T + \frac{Dk_T}{c_s c_p}\nabla C - \frac{1}{\rho c_P} q_y^r\right) \tag{1.16}$$

$$\boldsymbol{V}.\nabla C = \nabla.\left(D_e \nabla C + \frac{Dk_T}{T_{mf}}\nabla T\right) \tag{1.17}$$

where c_p, c_s are the specific heat at constant pressure and concentration respectively, k_T is the thermal diffusion and D is the solutal diffusivity. T_{mf} is the mean fluid temperature, α_e and D_e are the effective thermal and solutal diffusivities, respectively.

For sufficiently small isobaric changes in temperature and concentration, the fluid density depends linearly on temperature and concentration differences, which is called as the linear Boussinesq approximation and is given by

$$\rho = \rho_0[1 - \beta_T(T - T_0) - \beta_C(C - C_0)] \tag{1.18}$$

where ρ_0 is the fluid density at some reference point, with temperature and concentration as T_0, C_0 respectively and β_T is the coefficient of volumetric thermal expansion and β_C is the coefficient of volumetric concentration expansion.

1.3 Literature Review

Owing to the complex structure of the porous medium, idealized models are employed to understand the flow and transport phenomenon in the porous medium. None of the models till today could give the clear picture of the transport mechanism in porous medium, but still some of the mechanisms like radiation, stratification, thermal and solutal dispersion, viscous dissipation, melting and variable viscosity effects were described efficiently.

Porous materials are used in heat exchangers, building thermal insulators, porous insulators for fire fighting etc. In the nuclear waste disposal industry, to model a suitable canester is very essential for the safety analysis. Axi-symmetric bodies are utilized as canesters. Their disposal to the sea bed or to the earth's crust needs a better understanding of the convection phenomenon in the porous medium. The extraction of petroleum to the last drop from the oil reservoirs in the earth's crust needs a nice knowledge of the convection mechanism and thorough understanding of the miscible displacement techniques in porous medium.

With the occurrence of volcanism, magmatic intrusions may occur at the shallow depths in the earth's crust. Meteoric water, with which the earth is saturated, percolates down to the depth in the permeable formation and gets heated directly or indirectly by the intruded magma. Because of the density variations, this fluid is driven buoyantly upwards to the top of the aquifer. Hot fluids thus formed from aquifers continuously withdrawn by a down-hold pump. For geothermal power generation, the hot fluids are then piped to a geothermal power plant to drive the turbine directly or indirectly. It is understood that the geothermal energy can replace all other forms of energy but the identification of geothermal reservoir and extraction of geothermal energy needs more sophisticated technology.

One more interesting application of convection heat and mass transport in porous media is in the Resin Transfer Modeling (R T M). Resin is a non-Newtonian fluid. R T M is the process of producing fiber reinforced polymeric parts in final shape. Reinforced fiber is placed in a closed mold and resin is injected into the mold to fill up the pores. This is then cooled and cured and the fiber shapes are taken out from the mold in final form. With isothermal or non-isothermal resin filled into the pores, the cooling process needs an understanding of the convective phenomenon in fluid saturated porous medium. Some more interesting applications on heat and mass transfer in porous media saturated with power law fluids can be found in Shenoy (1994).

When the dimension of the convection system is large it has been the common practice in analytical studies to go for all possible simplifications with out loosing the physics of the problem. Boundary layer and Boussinesq approximations provide sufficient mathematical simplifications to tackle the problem analytically. Wooding (1963) was first to develop the boundary layer analogy while analyzing the steady vertical convection from a point source of heat in fluid saturated porous medium. He observed that the diffusion effects can be neglected except in regions where the gradients of fluid properties are vary large.

In order to solve the boundary layer equations, different methods such as similarity solution method, integral method, local similarity solution method, local non-similarity method, series solution technique finite difference method, finite element method and finite volume method are used. Similarity analysis is used to investigate the conditions under which the solutions of a particular boundary value problem have similar forms for different values of the independent variables. In two–dimensional flow problems, if similarity exists, then the independent variables can be merged into a single similarity variable and the governing partial differential equations are reduced into ordinary differential equations. This is a considerable mathematical simplification. The classical way of obtaining the similarity transformation (Sparrow et al. (1959)) is now replaced by the scaling approach (Bejan (2004)). Thus reduced ordinary differential equations allow the usage of the generalized techniques developed for solving ordinary differential equations. The motivation for seeking similar solutions is of three fold. Firstly, the results may be directly usable in important technical applications. Secondly, the similar solutions provide a standard comparison against which approximate methods may be checked. Once verified, the approximate methods may then be used in studying more complex flow situations where the conditions of similarity solutions are not satisfied. Finally, the general trends may provide valuable insight in understanding the physical occurrences which take place in complicated flows.

When similarity solution is not possible, one frequently used concept in the solution of non-similarity boundary layers is the principle of local similarity. In this approach, the relevant boundary layer equations suitably transformed and divested of non-similar terms is applied locally and independently at discrete stream-wise locations. The transformed and simplified equation resembles that for a similarity boundary layer and solved by well established techniques which gives the solution at the particular stream-wise location with out having to perform calculations at upstream locations. In local non-similarity method, local solutions are independent of upstream information but accounts for non-similarity of the boundary layer. The local similarity method and local non-similarity method are well described in Sparrow et al. (1970) and Minkowycz and Sparrow (1974).

Bejan (2004) discussed clearly the external and internal convection in Darcian fluid saturated porous medium. A very recent book by Nield and Bejan (2006) is a researcher's literature survey book giving every minute detail about the happenings in the field of convective heat and mass transfer in porous medium up to the year 2006. Cheng (1978) discussed on convective heat transfer aiming at the extraction of geothermal energy using governing equations in porous medium. In Sahimi (1995), the flow and transport in porous media and fractured rock has been discussed and various applications of the flow through porous media such as miscible displacement and immiscible displacement techniques are discussed. Johnson and Cheng (1977) reviewed all possible similarity solutions for free convection adjacent to flat plates which were dealt with the Darcy model. A more general similarity transformation was proposed by Nakayama and Koyama (1987a) for free convection as well as Nakayama and Koyama (1987b) for combined convection over a nonisothermal body of arbitrary shape embedded in a fluid saturated porous medium. In addition to the similarity analysis, integral techniques have also been employed by Nakayama and Koyama (1987a). These arbitrary shaped bodies include vertical flat plate, horizontal ellipse and ellipsoids with different minor-to-major axis ratios, vertical wedge, vertical cone, horizontal circular cylinder and a sphere.

Vafai and Tien (1981) analyzed the boundary and inertia effects on flow and heat transfer in porous media with constant porosity. It has been observed that the boundary effect is confined to a very thin momentum boundary layer and often plays an insignificant role in overall flow consideration. The inertia and boundary effects are found to be more pronounced in a highly permeable media, high Prandtl number fluids and in the region close to the leading edge of the flow boundary layer. A detailed discussion with boundary and inertia effects on convective mass transfer has been done by Vafai and Tien (1981).

Later Plumb and Huenefeld (1981) studied the non-Darcy natural convection over a vertical wall in a saturated porous medium. They used the Ergun model as the momentum equation. Unlike in the Darcy case, the similarity solution was possible only for isothermal wall. They used Darcy-buoyancy force comparison in the scale analysis and obtained the similarity transformation. Results reveal that the inertial effect thicken the boundary layer and resist the heat transfer. For the vertical heated surfaces the deviation of the heat transfer results from its Darcian counterpart is less than 5% for the modified Grashoff number less than 0.1 and when the modified Grashoff number is greater than 0.1, the deviation increases rapidly. Nakayama et al. (1990) discussed the Forchheimer free convection over a non-isothermal body of arbitrary shape in saturated porous medium. By considering inertia and buoyancy force comparison in scale analysis Bejan and Khair (1985), proposed new scales for boundary layer thickness and stream function in porous media flows. This analysis is effective at large Reynolds number limit where Darcy law is invalid and inertial effects are important. This transformation permits similarity solution for isothermal and isosolutal wall and also in the case with the surface heat and mass fluxes. Lai (1991) also analyzed the convective transport when the fluid is subjected to density variations as well as to an external flow field in porous medium.

To unveil the basics of natural convection along a vertical wall, so far many of the researchers have considered the simplest model possible, that is, the heat and mass transfer interaction between a vertical wall and an isothermal semi-infinite fluid reservoir. Proceeding now on the road from simple to the complex, we take a closer look at the problem of modeling a real situation involving natural convection. Vertical walls are rarely in communication with semi-infinite isothermal pools of fluid: more often, their height is finite and the heated boundary layer eventually hits the ceiling. At that point, the heated stream has no choice but to discharge horizontally into the fluid reservoir. The direction of discharge is horizontal because the discharge contains fluid warmer than the rest of the reservoir. The long-time effect of this discharge process is stratification, or warm fluid layers floating on top of gradually colder layers. Indeed, stratification is a characteristic of all fluid bodies surrounded by differentially heated sidewalls lined by buoyancy layers. At this point it is sufficient to recognize that the air in any room with the doors closed is thermally stratified in such a way that the lowest layers assume the temperature of the coldest wall, and the layers near the ceiling approach the temperature of the warmest wall.

Some special cases of thermal stratification in porous medium have been dealt by Bejan (2004), Nakayam and Koyama (1987c) and Takhar and Pop (1987). All these investigations are carried out for a Darcian porous medium. Angirasa and Srinivasan (1992) analyzed the effect of thermal stratification from a vertical wall in contact with a stratified reservoir using numerical finite-difference technique. Kalpana and Singh (1992) analyzed the problem of boundary layer free convection along an isothermal vertical plate immersed in a thermally stratified fluid saturated non-Darcy porous medium using series solution technique. The case of power law variation of wall temperature with thermal stratification of the medium was discussed at length by Nakayama and Koyama (1987c), and by Lai et al. (1990). The effect of thermal dispersion and stratification on mixed convection from a vertical surface in a Newtonian fluid saturated Darcy porous medium was investigated by Gorla et al. (1996). Later Hassanien et al. (1998) studied the same problem in a non-Darcy porous medium. Rathish Kumar and Shalini (2004) performed a numerical investigation on natural convection induced by a vertical wavy surface in a thermally stratified non-Darcy porous medium. The effects of variable viscosity on non-Darcy MHD free convection along a non-isothermal vertical surface in a thermally stratified porous medium were discussed by Afify (2007). More recently, Ishak et al. (2008) reported the mixed convection boundary layer flow over a vertical surface embedded in a thermally stratified porous medium. It was reported by Ishak et al. (2008) that the thermal stratification significantly affects the surface stress as well as the surface heat transfer, besides delaying the boundary layer separation.

At the moderate velocities, the thermal dispersion effects are observed to become prevalent. For the first time, dispersion effect was considered by Cheng (1981) and later by Plumb (1983) in the study of non-Darcy natural convection over a vertical flat plate. Thermal dispersion has components in longitudinal and transverse directions. Cheng (1981) assumed that dispersion coefficients are proportional to the velocity components and to the Forchheimer coefficients. He found that thermal dispersion decreases the surface heat flux. Plumb (1983) assumed that the longitudinal dispersion was negligible and the transverse dispersion was proportional to the stream-wise velocity components and gave the linear relation as $\alpha_d = \gamma du$ where γ is the mechanical dispersion coefficient which has to be determined from the experimental results, d is grain diameter. His expression for Nusselt number differs from that used in Cheng (1981). He observed that the inertial effect decreased the heat transfer where as thermal dispersion enhanced the same. Considering no slip boundary condition, Hong and Tien (1987) analyzed the analyzed the vertical plate natural convection in a non-Darcy porous medium. It was observed that due to the combination of no-slip and dispersion effects, the temperature gradient near the wall is increased.

Dagan (1972) analyzed the effect of double (thermal and solutal) dispersion in a Darcy porous medium. The effect of double dispersion in free convection boundary layer from the vertical wall is studied by Telles and Trevisan (1993) using scale analysis arguments in a Darcian porous medium. A detailed analysis regarding the effect of double dispersion on mixed convection heat and mass transfer in non-Darcy porous medium using similarity solution technique has been presented by Murthy (2000). It was reported that the flow, temperature and concentration fields are governed by the complex interaction among the diffusion rate, buoyancy ratio and flow deriving parameter. Double dispersion effect on natural convection heat and mass transfer in non-Darcy porous medium was investigated by El-Amin (2004). El-Hakiem (2001) analyzed the influence of thermal dispersion on Darcy free convection from a vertical wall in a non-Newtonian power fluid saturated porous medium with variable heat flux. In this thesis we analyzed the effect of double dispersion on natural and mixed convection heat and mass transfer from vertical plate in a power law fluid saturated porous medium. Also we studied the effect of thermal dispersion in a thermally stratified medium saturated with power law fluids.

But most of the engineering applications involve non-Newtonian fluids. An illustrative example is found in oil reservoir engineering in connection with the production of heavy crude oils which are power law fluids with yield stress. This process involves the cyclic injection steam into the well for the purpose of increasing the temperature of the oil reservoir, a procedure referred to as “steam soak” or “huff and puff” in the industry. The increase in the temperature of the reservoir decreases the fluid viscosity resulting in a substantial increase in the mobility of the heavy crude oil, thus improving the production flow rate by gravity drainage. It is obvious that the efficiency of this process can be increased by obtaining insight into the combined effects of convective heat and mass transfer and convective flow in a power law fluid filled porous medium. Environmental concerns have also led to an increased interest in flows of non-Newtonian fluids of power law behavior through porous media. This class of shear flow is currently encountered in environmental protection. An example in the environmental context is where waste fluids containing contaminants may penetrate ground-water reservoirs. These pollutants, produced particularly in chemical industries, can spread as gravity currents if the density of the contaminated fluid is different from that of the ground-water. For this reason, contaminated fluids must be stored in “safety” reservoirs, as for example depleted oil in aquifer reservoirs. As in the previous examples, non-Newtonian shear flows are so widespread in industrial processes and the environment that it would be no exaggeration to affirm that Newtonian shear flows are the exception rather than the rule. Shenoy (1994) presented many interesting applications of non-Newtonian power law fluids with yield stress convective heat transport in fluid saturated porous media considering geothermal engineering applications and oil reservoir engineering applications.

Chen and Chen (1988a) were the first to consider the simplest free convection flow of non-Newtonian fluid past an isothermal vertical flat plate embedded in a porous medium. Their analysis was later extended (Chen and Chen (1988b)) to include other body shapes such as horizontal cylinders and spheres. Nakayama and Koyama (1991) analyzed the more general case of free convection over a non-isothermal body of arbitrary shape embedded in a non-Newtonian fluid saturated porous medium. The boundary layer flow and heat transfer of non-Newtonian fluid in porous media was investigated by Chaoyang and Chaunjing (1989). Combined free- and forced-convection heat transfer to power law fluid saturated porous media was analyzed by Nakayama and Shenoy (1992) and similarity solutions were presented for vertical plates, cones, horizontal cylinders and spheres. The modified form of the Darcy-Forchheimer equation for non-Newtonian power law fluids has been developed by Shenoy (1993). Using the proposed equation for mathematically describing non-Darcy flows, Shenoy (1993) analyzed the various cases of Darcy-Forchheimer regimes systematically and compared the results of the analysis with existing exact and approximate solutions to show an agreement with in 2.5% of reported exact values. A unified similarity transformation procedure along the similar lines of Nakayama and Pop (1989) for Darcy and non-Darcy forced, free and mixed convection heat transfer in non-Newtonian inelastic power law type fluids was studied by Nakayama and Shenoy (1992).

Rastogi and Poulikakos (1995) examined the problem of double diffusive convection from a vertical plate in a porous medium saturated with a non-Newtonian power law fluid. They concluded that the effect of the Lewis number and buoyancy ratio on the surface heat and mass flux is more prominent for pseudoplastic fluids. Mixed convection in non-Newtonian fluids along a vertical plate in a porous medium was investigated by Gorla and Kumari (1996). They pointed that the heat transfer rate for pure forced convection is greater than that for pure free convection. Jumah and Mazumdar (2000) studied the free convection heat and mass transfer of a non-Newtonian power law fluid with yield stress from a vertical flat plate in a saturated porous medium. Ibrahim et al. (2000) studied the non-Darcy mixed convection flow along a vertical plate embedded in a non-Newtonian fluid saturated porous medium with surface mass transfer. Chamkha and Al-Humoud (2007) examined the problem of mixed convection heat and mass transfer of non-Newtonian fluids from permeable surface embedded in a porous medium in the presence of suction or injection and heat generation or absorption effects. Similarity solution for the natural convection heat transfer of non-Newtonian fluids in porous media from a vertical cone under mixed thermal boundary conditions was obtained by Cheng (2009).

Viscous dissipation is of particular significance in natural convection in various devices that are subjected to large variation of gravitational force or that operate at high rotational speeds, Gebhart (1962) pointed out in his study of viscous dissipation on natural convection in fluids. Similarity solution for the same problem with exponential variation of wall temperature was obtained by Gebhart and Mollendor (1969). Fand et al. (1986) observed from their experiment that the effect of viscous dissipation is significant when the saturating fluid is silicon oil. Using the one dimensional flow model and incorporating the viscous dissipation effects in the energy equation they observed that a decrease in the value of Nusselt number results. More recently, Nakayama and Pop (1989) analyzed the effect of viscous dissipation on the free convection over a non-isothermal body in a porous medium. They used the Karman-Pohlhausen integral method in their study. It was observed that the viscous dissipation lowers the level of heat transfer rate.

Murthy and Singh (1997a) studied viscous dissipation on non-Darcy natural convection regime in porous media saturated with Newtonian fluid. A fall in the heat transfer rate was observed with the introducing of viscous dissipation effect. The effect of viscous dissipation on the development of the boundary layer flow from a vertical surface in a Darcian porous medium was investigated by Rees et al. (2003). They found that the flow evolves gradually from the classical Cheng-Minkowycz (1977) form to asymptotic dissipation profile which is parallel flow. A critical review of the modeling of viscous dissipation and its alternatives forms in a saturated porous medium was presented by Nield (2007). Many non-Newtonian liquids are highly viscous such that the irreversible work due to viscous dissipation can, in some instance, becomes quite important which motivated us to study the viscous dissipation phenomena in non-Newtonian fluid saturated porous medium. Along this line El-Amin et al (2003) investigated the problem of viscous dissipation on buoyancy induced flow over (a horizontal and vertical flat plate) embedded in a non-Newtonian fluid saturated porous medium. Later their analysis was extended by El-Amin (2003) combing the effect of MHD and viscous dissipation with considering variable surface heat flux.

It has been found that an energy flux can be generated not only by temperature gradient but by concentration gradient as well. The energy flux caused by a concentration gradient is called the Dufour or diffusion–thermo effect. On the other hand, a mass flux created by temperature gradients is the Soret or thermo–diffusion effect. These effects become significant when the temperature and concentration gradients are very large. Even though Soret and Dufour effects are considered as second order phenomenon, these effects are significant in hydrology when studying mineral enrichment of geothermal sources, in petrology when investigating hydro carbon segregation, in geosciences when considering magma differentiation, etc. Luikov and Mikhailov (1936) discussed the Soret effect in drying processes where the migration of moisture is caused by the temperature gradient. Hurle and Jackeman (1971) argued that for liquid mixture the Dufour term is indeed very small and thus the Dufour effect may be negligible in comparison to the Soret effect. Benano-Melly et al. (2001) considered thermal diffusion induced by horizontal thermal gradient in a binary mixture with in a porous medium. Bourich et al. (2004) studied analytically and numerically the Soret effect on the onset of convection in vertical porous layer subjected to uniform heat fluxes.

An analytical study of linear and non-linear double diffusive convection in couple stress liquids with Soret effect reported by Malashetty et al. (2006). Postelnicu (2007) studied numerically the influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects. Thermal diffusion and MHD effects on combined, free-forced convection and mass transfer of a viscous fluid flow through a porous medium with heat generation is examined by Abdel-Rahman (2008). The linear stability analysis of Soret-driven thermo-solutal convection in a shallow horizontal layer of a porous medium subjected to inclined thermal and solutal gradients of finite magnitude is investigated theoretically by Narayana et al. (2008). The significance of the Soret effect motivated us to consider this effect in a power fluid saturated non-Darcy porous medium.

Depending on the surface properties and geometry, radiative heat transfer is often comparable to or larger than the convective heat transfer in many practical applications. Using Beer's law of radiation absorption, steady laminar two dimensional free convection flow of an absorbing fluid up a vertically heated plate embedded in a porous medium has been studied by Chamkha (1997) with a more general Darcy–Forchheimer–Brinkman flow model. Using Rosseland diffusion approximation for thermal radiation heat flux Raptis (1998) studied the simultaneous free convection and thermal radiation in a fluid saturated porous medium. This has been followed by several researchers to study convective heat transfer coupled with thermal radiation. While discussing radiative heat transfer in fibrous Tong and Tien (1983) have shown that the thermal radiations are very important even in a low temperature. Also they reported that it could account for as much as 30 percent of total heat transfer.

Thermal radiation on buoyancy induced flow of power law fluids over a horizontal plate of variable temperature was examined by Mohhammadein and El-Amin (2000). They reported that radiation enhances the heat transfer rate while power law index parameter reduces the same. Murthy et al. (2004) analyzed both the aiding and opposing flows for the combined radiation and mixed convection from permeable vertical wall in a non-Darcy porous media. It has been reported that radiation is prominent in the near Darcy region than in the non-Darcy region. The effect of Radiation and thermal dispersion on non-Darcy natural convection with lateral mass flux for non-Newtonian power law fluid from a vertical flat plate saturated in porous medium was examined by Ibrahim et al. (2005).

Several studies on convection in porous media have been reported by considering constant physical properties of the ambient fluids. However, it is well known that the viscosity of liquid changes evidently with the temperature and this influences the variation of velocity through the flow. Therefore, in practical heat transfer problems with large temperature difference between the surface and the fluid, considering constant viscosity might lead to a considerable error in assessing the heat transfer coefficient. Considering variable viscosity effect, Lai and Kulacki (1990) studied the mixed convection flow along a vertical plate embedded in a porous medium. A theoretical study of temperature dependent viscosity for the forced convection flow thorough the semi-infinite porous medium bounded by an isothermal plate presented by Ling and Dybbs (1992) with considering viscosity as inverse function of temperature. Later, Postelnicu et al. (2001) extended their work by considering internal heat generation in the medium. The problem of variable viscosity on non-Darcy free or mixed convection flow on a vertical surface in a Newtonian and non-Newtonian fluid saturated porous medium examined by Jayanthi and Kumari (2006, 2007). They pointed that in liquids the heat transfer is more in case of variable viscosity than the case of constant viscosity where as for gases reversed trend was observed. Elbashbeshy (2000) numerically investigated the MHD free convection flow variable viscosity and thermal diffusivity along a vertical plate. The Reynolds viscosity model was used to characterize the variation in viscosity. Flow of a generalized second grade non-Newtonian fluid with viscosity varying exponentially with temperature was studied by Massoudi and Phuoc (2004).

Phase change in convective transport in porous media finds importance in applications in magma solidification and the melting of permafrost. In this direction, Epstien and Cho (1976) considered the laminar film condensation on a vertical melting surface for 1-D and 2-D system based on Nusselt's method to discuss the melting rate. They concluded that as long as melting solid is large compared with the thickness of thermal boundary layer, the transient effects in the solid would be neglected. Sparrow et al. (1977) studied the velocity and temperature fields, heat transfer rate, melting layer thickness by means of finite difference scheme in the melting region for natural convection, while Kazmierczak et al. (1986, 1987) analyzed melting from a vertical flat plate embedded in a porous medium in both free and forced convection processes. The heat transfer at the melting surface in the laminar boundary layer was discussed by Pozvonkov et al. (1970) using Karman-Pohlhausen method. Poulikakos and Spatz (1988) analyzed the melting phenomena on free convection from a vertical front in a non-Newtonian fluid saturated porous matrix. It was concluded that a non-Newtonian melt may yield faster (for dilatants) or slower (for pseudoplastics) than a Newtonian melt.

Bakier (1997) studied the melting effect on mixed convection from a vertical plate of arbitrary wall temperature both in aiding and opposing flows in a fluid saturated porous medium while Gorla et al. (1999) considered similar study with uniform wall temperature conditions. Both of these studies showed that there is a significant fall in the heat transfer rate at the solid liquid interface. Tashtoush (2005) examined the magnetic and buoyancy effects to investigate the flow, temperature profiles and heat transfer characteristics for melting effect associated with uniform wall temperature based on non-Darcy flow model. The melting phenomena on unsteady and steady mixed convection heat transfer from a vertical plate in a liquid saturated porous medium with aiding and opposing external flows has been studied by Cheng and Lin (2006, 2007). Later they extended their work to the mass transfer (2008) by considering double diffusion processes. In this thesis we analyzed the melting effect on natural and mixed convection heat and mass transfer in power law fluids from a vertical plate with considering Soret effect in the medium.

1.4 Overview of the Thesis

The work done in this thesis can be divided in to two parts, through chapters 2 and 3, we analyzed different flow, temperature and heat transfer characteristics of the non-Newtonian power law fluids with second order effects such as thermal stratification, thermal dispersion, viscous dissipation, variable viscosity and thermal radiation. In chapters 4 to 8 coupled heat and mass transfer on free and mixed convection with a choice of some second order effect has been analyzed

In Chapter 1, an introduction to various flow models that are used in porous media is given along with a discussion on various second order effects which influence the flow filed and convective transport in the medium. A brief survey of literature on Newtonian and non-Newtonian flows through porous media is also presented. Effect of thermal dispersion on free convective heat transfer from a vertical flat plate in a power law fluid saturated thermally stratified non-Darcy porous medium is presented in Chapter 2. The similarity solution is possible when the wall temperature and medium stratification are assumed to have a specific power function form. Main focus in Chapter 3 is to examine the combined effects of viscous dissipation and radiation on natural convection heat transfer from vertical flat plate in a non-Darcy porous medium saturated with power law fluid of variable viscosity. The wall and the ambient medium are maintained at constant but different levels of temperature. The viscosity of the fluid is assumed to follow Reynolds viscosity model. Rosseland approximation is used to describe the radiative heat flux in the energy equation. The non-similar equations are solved numerically by local non-similarity method.

The influence of melting and thermo-diffusion (Soret) effect on natural and mixed convection heat and mass transfer from vertical flat plate is presented in Chapter 4 and Chapter 5, respectively. The wall and the ambient medium are maintained at constant but different levels of temperature and concentration such that the heat and mass transfer occurs from the wall to the medium. The effect of power law index parameter on natural convection heat and mass transfer from a vertical wall is analyzed by considering double dispersion with constant wall temperature and concentration conditions is presented in

Chapter 6 while the same for the mixed convection is presented in Chapter 7. In Chapter 8 we present the influence of viscous dissipation and Soret effect on natural convection heat and mass transfer from vertical cone/flat plate. The surface of the cone and the ambient medium are maintained at constant but different levels of temperature and concentration. The governing equations are non-dimensionalized into non-similar form and then solved numerically by local non-similarity method.